

CONTENTS

1	z-TRANSFORM DEFINITION AND THEOREMS	1
1.1	Discrete Time Function and z-Transform Definitions	2
1.2	Properties of z-Transforms	3
1.3	Inverse z-Transform and Branch Points	9
1.4	The Modified z-Transform	15
1.5	Relationship between Laplace and z-Transforms	20
1.6	Application to Sampled-Data Systems	28
1.7	Mean Square Value Theorem	29
1.8	Equivalence between Inverse Laplace and Modified z-Transforms	31
1.9	Other Transform Methods	36
	Appendix. A Method of Determining the Coefficients of the z-Transform Expansion	41
2	z-TRANSFORM METHOD OF SOLUTION OF LINEAR DIFFERENCE EQUATIONS	45
2.1	Linear Difference Equations with Constant Coefficients	45
2.2	Solution of Difference Equations Whose Coefficients Are Periodic Functions	48
2.3	Linear Difference-Differential Equations	51
2.4	Difference Equations with Periodic Coefficients	57
2.5	Time-Varying Difference Equations	59
2.6	Time-Varying z-Transform and the System Function	66
2.7	Double z-Transformation and Solution of Partial Difference Equations	73

3	STABILITY CONSIDERATION FOR LINEAR DISCRETE SYSTEMS	79
3.1	Definition of Stability	80
3.2	Stability Condition for Linear Time-Varying Discrete Systems	81
3.3	Tests for Stability	82
3.4	Stability Test Directly Applied in the z -Plane	84
3.5	Determinant Method	85
3.6	Critical Stability Constraints for System Design	94
3.7	Number of Roots of a Real Polynomial Inside the Unit Circle	95
3.8	Relationship between the Determinant Method and Hurwitz Criterion	97
3.9	Table Form	97
3.10	Division Method	109
3.11	Aperiodicity Criterion for Linear Discrete Systems	112
3.12	Theorems Related to Stability and Number of Roots	116
	Appendices	
	1. Derivation of the Table Form of Stability	121
	2. Singular Cases in Determinant and Table Forms	128
	3. Summary of the Stability Criteria	136
4	CONVOLUTION z-TRANSFORM	142
4.1	Complex Convolution Theorem	142
4.2	Complex Convolution Theorem for the Modified z -Transform	145
4.3	Applications of the Convolution Modified or z -Transform Method	147
	Appendices	
	1. Proof of Complex Convolution Formula	167
	2. Derivation of Total Square Integrals Formula	168
5	CONVOLUTION z-TRANSFORM APPLIED TO NONLINEAR DISCRETE SYSTEMS	174
5.1	Assumptions	175
5.2	Convolution z -Transforms of Certain Functions	176
5.3	Method of Solution for Second- and Higher-Order Equations	177
5.4	Illustrative Examples	179
6	PERIODIC MODES OF OSCILLATION IN NONLINEAR DISCRETE SYSTEMS	189
6.1	Limit Cycle Analysis of Nonlinear Discrete Systems	189
6.2	Application of the Fundamental Equation to Specific Examples	196

6.3	Limitation on the Period of Limit Cycles of Relay Mode Oscillations	203
6.4	Stability Study of Limit Cycles	207
6.5	Forced Oscillations in Nonlinear Discrete Systems	208
6.6	Direct z -Transform for Determining True Oscillation	213
6.7	Periodic Solution of Certain Nonlinear Difference Equations	215
7	z -TRANSFORM METHOD IN APPROXIMATION TECHNIQUES	219
7.1	Approximation Methods	219
7.2	Initial Conditions Nonzero	223
7.3	Integrating Operators	229
7.4	z -Forms and Modified z -Forms	231
7.5	The Choice of the Sampling Period	238
7.6	Analysis of the Error	239
7.7	Low-Pass Transformation for z -Transforms	240
7.8	Applications to Time-Varying Differential Equations	241
7.9	Application to Nonlinear Differential Equations	243
7.10	Other Numerical Techniques	246
8	APPLICATIONS TO VARIOUS AREAS OF SYSTEM THEORY	248
8.1	Nonlinear Sampled-Data Feedback Systems	248
8.2	Analysis of Discrete Antenna Array by z -Transform Method	254
8.3	Application to Information and Filtering Theory	258
8.4	z -Transform Method Applied to Problems of Economics	263
8.5	Linear Sequential Circuits	267
8.6	Application to Discrete Markov Processes	270
APPENDIX		
	Table I z -Transform Pairs	278
	Table II Pairs of Modified z -Transforms	289
	Table III Total Square Integrals	297
	Table IV Closed Forms of the Function $\sum_{n=0}^{\infty} n^r x^n, \quad x < 1$	300
PROBLEMS		
		301
INDEX		
		321