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Part One

NUMERICAL INVERSION OF LAPLACE TRANSFORMS

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Part Two
NUMERICAL TABLES

Table 1. TABULAR POINTS AND WEIGHTS FOR FORMULA OF MAXIMUM DEGREE OF PRECISION

$$\frac{1}{2\pi i} \int_{\varepsilon-i\infty}^{\varepsilon+i\infty} e^p p^{-s} \varphi(p) dp \approx \sum_{k=1}^n A_k \varphi(p_k)$$

FOR $s = 1, 2, 3, 4, 5$; $n = 1(1)15$, TO 20 PLACES 49

Table 2. TABULAR POINTS AND WEIGHTS FOR FORMULA OF MAXIMUM DEGREE OF PRECISION

$$\frac{1}{2\pi i} \int_{\varepsilon-i\infty}^{\varepsilon+i\infty} e^p p^{-s} \varphi(p) dp \approx \sum_{k=1}^n A_k \varphi(p_k)$$

FOR $s = 0, 01(0,01)3$, $s \neq 1, 2, 3$; $n = 1(1)10$, TO 7-8 PLACES . . . 63

Table 3. TABULAR POINTS FOR EQUALLY-WEIGHTED FORMULA

$$\frac{1}{2\pi i} \int_{\varepsilon-i\infty}^{\varepsilon+i\infty} e^p p^{-s} \varphi(p) dp \approx \frac{1}{n \Gamma(s)} \sum_{k=1}^n \varphi(p_k)$$

FOR $s = 1$, $n = 1(1)10$, TO 12 PLACES 263

Table 4. COEFFICIENTS a_{kj} FOR EVALUATION OF QUADRATURE WEIGHTS $A_k(t)$ IN NUMERICAL FORMULAS FOR FOURIER TRANSFORMS AND INVERSE LAPLACE TRANSFORMS. EQUIDISTANT TABULAR POINTS, $n = 1(1)15$, TO 10 PLACES 265

Table 5. TABLE OF c_{kj} FOR QUADRATURE FORMULA

$$f(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{pt} p^{-1} \varphi(p) dp \approx \sum_{k=0}^n \left\{ \sum_{j=0}^n c_{kj} t^j \right\} \frac{\varphi(k+1)}{k+1},$$

$n = 1(1)9$, TO 12 PLACES 273

Table 6. COEFFICIENTS a_{kj} FOR EVALUATION OF QUADRATURE WEIGHTS $A_k(t)$ IN NUMERICAL FORMULAS FOR FOURIER TRANSFORMS AND INVERSE LAPLACE TRANSFORMS. TABULAR POINTS $\sigma_k = \frac{1+x_k}{1-x_k}$, x_k THE ROOTS OF THE CHEBYSHEV POLYNOMIALS OF THE FIRST KIND, $n = 1(1)14$, TO 10 PLACES 276

Table 7. COEFFICIENTS a_{kj} FOR EVALUATION OF QUADRATURE WEIGHTS $A_k(t)$ IN NUMERICAL FORMULAS FOR FOURIER TRANSFORMS AND INVERSE LAPLACE TRANSFORMS. TABULAR POINTS $\sigma_k = \frac{1+x_k}{1-x_k}$, x_k THE ROOTS OF THE LEGENDRE POLYNOMIALS, $n = 1(1)14$, TO 10 PLACES 285