

Contents

Preface	XIII
Chapter 1. Linear operators in L_α spaces	1
1. <i>The space L_α</i>	1
1.1. Description of the spaces	
1.2. Criteria for compactness	
1.3. Continuous linear functionals and weak convergence	
1.4. Semi-ordering in the spaces S and L_α	
1.5. Projections and bases of Haar type	
1.6. Operators in the spaces L_α	
2. <i>Continuous linear operators</i>	17
2.1. Linear operators	
2.2. Regular operators	
2.3. The M. Riesz interpolation theorem	
2.4. Interpolation theorems for regular operators	
2.5. Classes of L -characteristics of linear operators	
2.6. On a property of regular operators	
2.7. The Marcinkiewics interpolation theorem	
3. <i>Compact linear operators</i>	48
3.1. Compact linear operators	
3.2. Compactness and adjoint operators	
3.3. Properties of operators compact in measure	
3.4. Interpolation properties of compactness	
3.5. Strongly continuous linear operators	
Chapter 2. Continuity and compactness of linear integral operators	64
4. <i>General theorems on continuity on integral operators</i>	64
4.1. Linear integral operators	
4.2. Regular operators	

Contents

4.3. Example of a non-regular operator	
4.4. The adjoint operator	
4.5. Operators with symmetric kernels	
4.6. Products of integral operators	
4.7. Truncations of kernels of integral operators	
5. <i>General theorems on compactness of integral operators</i>	81
5.1. Problem setting	
5.2. Regular operators acting from L_0 to L_{β_0} and from L_{α_0} to L_1	
5.3. Regular operators acting from L_{α_0} to L_{β_0} , where $0 < \alpha_0 < 1$, $0 < \beta_0 \leq 1$	
5.4. Regular operators acting from L_{α_0} to L_{β_0} , where $0 < \alpha_0 < 1$, $\beta_0 \geq 1$	
5.5. Regular integral operators acting from L_1 to L_{β_0}	
5.6. Operators with compact majorants	
5.7. The case of kernels with reinforced singularities	
5.8. Truncations of kernels of integral operators	
5.9. Products of integral operators	
5.10. Compactness of non-regular operators	
6. <i>Linear u_0-bounded operators</i>	97
6.1. Simplest criteria for continuity of integral operators	
6.2. Spaces E_{u_0}	
6.3. General form of u_0 -bounded operators	
6.4. Compactness of u_0 -bounded operators	
6.5. Compactness of u_0 -bounded operators acting from L_α to L_0	
6.6. Compactness of u_0 -bounded operators acting from L_1 to L_β ($\beta > 0$)	
6.7. Integral operators acting from L_α to C	
6.8. v_0 -Cobounded linear operators	
6.9. Compactness of v_0 -cobounded operators	
6.10. Interpolation properties of u_0 -boundedness	
6.11. On weakly compact operators in L_1	

7. <i>Integral operators with kernels satisfying conditions of Kantorovic type</i>	118
7.1. Simplest criteria	
7.2. Theorems with intermediate conditions	
7.3. Lemmas	
7.4. Applications of theorems on adjoint operators	
7.5. Fundamental theorems	
7.6. Conditions of 'Kantorovic' type	
7.7. Summability of kernels of integral operators	
8. <i>Operators of potential type</i>	144
8.1. Definitions	
8.2. Simplest theorems on continuity and compactness of potentials	
8.3. Interpolation theorem of Stein-Weiss	
8.4. Limit theorems on continuity of potentials	
8.5. Operators of potential type	
8.6. The logarithmic potential	
8.7. Iterates of operators of potential type	
8.8. Generalizations to the case of distinct dimensions	
8.9. Potentials with respect to non-Lebesgue measure	
Chapter 3. Fractional powers of selfadjoint operators	173
9. <i>Splitting of linear operators</i>	173
9.1. Square root of selfadjoint operators	
9.2. Splitting of an operator	
9.3. <i>L</i> -Characteristic of a square root	
9.4. Representation of compact operators	
9.5. Square root of integral operator	
9.6. Example	
9.7. Investigation of integral operators by means of properties of iterated kernels	
9.8. Remark on Mercer's theorem	
10. <i>Fractional powers of bounded operators</i>	191
10.1. The spectral function	
10.2. Fractional powers of bounded selfadjoint operators	

Contents

10.3. The fundamental theorem	
10.4. Operators in real spaces	
10.5. Fractional powers of compact operators	
10.6. <i>L</i> -Characteristics of fractional powers of operators	
10.7. Fractional powers of integral operators	
11. <i>Unbounded selfadjoint operators</i>	205
11.1. Closed operators	
11.2. Adjoint operators	
11.3. Integration with respect to spectral functions	
11.4. The fundamental theorem on spectral representation of unbounded selfadjoint operators	
11.5. Functions of selfadjoint operators	
11.6. Commuting selfadjoint operators	
11.7. Integrals of operator-functions	
11.8. Integral representation of fractional powers of an operator	
12. <i>Properties of fractional powers of unbounded operators</i>	222
12.1. Problem setting	
12.2. The moment inequality for fractional powers	
12.3. Subordinate operators	
12.4. Subordination of fractional powers	
12.5. Heinz' first inequality	
12.6. Heinz' second inequality	
12.7. Fractional powers of projected operators	
12.8. On a special class of selfadjoint operators	
12.9. Theorems on splitting	
12.10. Theorems on fractional powers	
12.11. <i>L</i> -Characteristic of fractional powers	
Chapter 4. Fractional powers of operators of positive type	248
13. <i>Semi-groups of operators</i>	248
13.1. Vector-functions and operator-functions	
13.2. Unbounded operators	
13.3. Resolvents	

13.4.	Definition of a semi-group	
13.5.	Generator of a semi-group	
13.6.	Theorem of Hille-Phillips-Miyadera	
13.7.	Analytic-semi-groups	
13.8.	Estimates for the operators $A^n T(t)$	
14.	<i>Fractional powers of positive-type operators</i>	279
14.1.	Positive-type operators	
14.2.	Negative fractional powers	
14.3.	Positive fractional powers	
14.4.	A moment inequality	
14.5.	Operators subordinate to fractional powers of a positive-type operator	
14.6.	General theorems on subordination	
14.7.	Estimates for elements of the form $BA^{-\tau}x$	
14.8.	Comparison of fractional powers of two operators	
14.9.	Fractional powers of positive-type generators	
14.10.	Compactness of fractional powers	
14.11.	Supplementary remarks	
15.	<i>Moment inequalities and L-characteristics of fractional powers</i>	306
15.1.	Lorentz spaces	
15.2.	Linear operators	
15.3.	Interpolation theorems	
15.4.	Fundamental theorems	
15.5.	L-Characteristics of fractional powers	
15.6.	One more theorem on compactness	
16.	<i>Fractional powers of elliptic operators</i>	324
16.1.	Elliptic differential expressions	
16.2.	Elliptic operators	
16.3.	Positive-type elliptic operators	
16.4.	Multiplicative inequalities and fractional powers of elliptic operators	
16.5.	L-Characteristics of negative fractional powers of elliptic operators	

Contents

16.6. Further theorems	
16.7. On integral representations of fractional powers of elliptic operators	
Chapter 5. Non-linear integral operators	349
17. <i>The superposition operators</i>	<i>349</i>
17.1. On functions which are continuous in one variable	
17.2. Simplest properties of the superposition operator	
17.3. Fundamental theorems	
17.4. Examples	
17.5. General form of L -characteristics of superposition operators	
17.6. Uniform continuity of the superposition operator	
17.7. Improvement of superposition operators	
17.8. Supplementary remarks	
18. <i>Conditions for continuity of integral operators</i>	<i>373</i>
18.1. Definitions and simple properties	
18.2. Conditions for continuity of Uryson operators	
18.3. General theorem on continuity of Uryson operators	
18.4. On a property of Uryson operators	
18.5. Regular Uryson operators	
18.6. Special examples	
18.7. Uryson operators with values in the space of bounded functions	
18.8. On uniform continuity of Uryson operators	
19. <i>Conditions for complete continuity of an Uryson operator</i>	<i>398</i>
19.1. Problem setting	
19.2. Hammerstein operators	
19.3. Complete continuity of regular Uryson operators acting from L_0 to L_β , $\beta \in (0, 1]$	
19.4. Complete continuity of regular Uryson operators acting from L_α to L_β , $\alpha > 0$, $0 \leq \beta \leq 1$	
19.5. Special criteria for complete continuity	
19.6. On L -characteristics of Uryson operators	

19.7.	Weakening of singularities	
19.8.	On two criteria for compactness (in measure) of operators	
19.9.	Complete continuity of Uryson operators with values in L_0	
20.	<i>Differentiation of non-linear operators</i>	417
20.1.	Derivative of a non-linear operator	
20.2.	General form of the derivative of a superposition operator	
20.3.	Conditions for the differentiability of a superposition operator on the whole space	
20.4.	Sufficient criteria for the differentiability of a superposition operator	
20.5.	Differentiability of superposition operators on dense sets	
20.6.	Derivatives of Hammerstein operators	
20.7.	Derivatives of Uryson operators	
20.8.	A general theorem	
20.9.	Partial criteria for differentiability of Uryson operators	
20.10.	Differentiability of Uryson operators at distinguished points	
20.11.	Asymptotic derivatives of non-linear operators	
20.12.	On higher order derivatives	
Chapter 6.	Some applications	450
21.	<i>Equations with completely continuous operators</i>	450
21.1.	Linear equations	
21.2.	On approximate solutions of equations	
21.3.	Existence of solutions of non-linear integral equations	
21.4.	Eigenfunctions of non-linear integral operators	
22.	<i>Convergence of Fourier's method</i>	463
22.1.	General theorems on convergence of Fourier's method	
22.2.	Convergence of Fourier series with respect to eigenfunctions of elliptic operators	
22.3.	Fourier's method for hyperbolic equations	
22.4.	Fourier's method for parabolic equations	

Contents

23. <i>Translation operators along trajectories of differential equations</i> . . .	475
23.1. Linear equations	
23.2. The Cauchy operator	
23.3. Non-linear equations	
23.4. Equations with unbounded non-linearities	
23.5. The translation operator	
23.6. Differentiability of the translation operator	
23.7. The quasi-translation operator	
23.8. Equations with variable operators	
23.9. The translation operator and periodic solutions of parabolic equations	
Bibliography	502
Index of terminologies	514
Index of notations	517
Author index	519