

Table of Contents

Introduction	11
Part I: Asymptotic Expansions	
1. Asymptotic Systems and Expansions	15
1.1 Basic Definitions and Examples	15
1.2 Problems	19
2. Geometric Asymptotic Expansions	21
2.1 Asymptotic Behavior of Iteration Processes	22
2.2 Eigenvalue Expansions	29
2.2.1 One-Step Matrix Iteration	29
2.2.2 The Power Method	31
2.3 Problems	37
3. Logarithmic Asymptotic Expansions	39
3.1 Some Introductory Examples	40
3.1.1 Numerical Computation of π	40
3.1.2 Continued Fractions of Poincaré Type	41
3.1.3 Numerical Differentiation	43
3.1.4 Asymptotic Expansions for the Error of Best Uniform Polynomial Approximation	44
3.2 Euler's Summation Formula and Riemann's Zeta-Function	47
3.2.1 Bernoulli Polynomials	48
3.2.2 Euler's Summation Formula	52
3.2.3 Asymptotic Expansion for the Partial Sums of Riemann's Zeta-Function	55
3.2.4 Generalization to Other Infinite Series	58
3.3 Asymptotic Expansions with a Prescribed Limit Function	61
3.3.1 Partial Products of Infinite Products	61

3.3.2 Use of Taylor's Expansion	65
3.3.3 Elliptic Functions and Elliptic Integrals	71
3.4 Numerical Quadrature Rules	76
3.4.1 The "Classical" Case: Sufficiently Smooth Integrands	76
3.4.2 Integrands with an Endpoint Singularity	82
3.4.3 Integrands with a Singularity in the Interior of the Integration Range	93
3.4.4 Multivariate Integration	101
3.5 Divided Differences and Numerical Differentiation. The E-Method	103
3.5.1 Classical Divided Differences	104
3.5.2 Generalized Divided Differences	109
3.5.3 The E-Method	118
3.6 Discretization Methods for Ordinary Differential Equations	128
3.6.1 One-Step Methods for First-Order Problems	130
3.6.2 Runge-Kutta Methods and Generalized Trapezoidal Rules	140
3.6.3 Linear Multistep Methods	153
3.6.4 Existence of Asymptotic Expansions	163
3.6.5 Miscellanea	180
3.7 Problems	185

Part II: Linear Extrapolation Methods

4. Fundamental Concepts and General Philosophy	193
4.1 Geometric Extrapolation	194
4.1.1 The General Situation	194
4.1.2 Extrapolation of Special Geometric Asymptotic Expansions	200
4.1.3 Extrapolation of Supergeometric Asymptotic Expansions	205
4.2 Logarithmic Extrapolation	208

4.2.1 Extrapolation of Logarithmic Asymptotic Expansions: The Procedure	209
4.2.2 Numerical Examples	211
4.2.3 The Name "Extrapolation"; Generalization to Other Index Sequences	216
4.3 Some Specific Numerical Methods as Special Cases of Logarithmic Extrapolation	224
4.3.1 Romberg Quadrature	224
4.3.2 Richardson Extrapolation and Gragg's Method	234
4.3.3 Numerical Computation of the Matrix Exponential	240
4.4 Problems	241
5. Error Bounds, Stopping Rules and Monotonicity	244
5.1 Asymptotic Error Bounds and Inclusions	244
5.1.1 Some Remarks on Exact Error Bounds	245
5.1.2 Asymptotic Error Bounds for Logarithmic Extrapolation	247
5.2 Error Bounds and Stopping Rules for Normal Expansions	257
5.2.1 A-Priori Error Bounds	260
5.2.2 Stopping Rules	262
5.2.3 Examples	268
5.3 Miscellanea	279
5.3.1 Propagation of Rounding Errors	279
5.3.2 Monotonicity Preserving Extrapolation	283
5.4 Problems	287
6. Generalizations and Final Remarks	289
6.1 Triangular Recursion Schemes	289
6.2 A Quite General Extrapolation Algorithm	296
6.3 Problems	302
Historical Notes	303
References	308
Index	329