Contents

Preface xi
Notation and Terminology xiv

Chapter 1. Elementary Theory of Holomorphic Functions 1
1. Some basic properties of $\mathbb{C}$-differentiable and holomorphic functions 2
2. Integration along curves 9
3. Fundamental properties of holomorphic functions 21
4. The theorems of Weierstrass and Montel 32
5. Meromorphic functions 36
6. The Looman-Menchoff theorem 43

Notes on Chapter 1 50
References: Chapter 1 51

Chapter 2. Covering Spaces and the Monodromy Theorem 53
1. Covering spaces and the lifting of curves 53
2. The sheaf of germs of holomorphic functions 55
3. Covering spaces and integration along curves 58
4. The monodromy theorem and the homotopy form of Cauchy's theorem 60
5. Applications of the monodromy theorem 64

Notes on Chapter 2 69
References: Chapter 2 69

Chapter 3. The Winding Number and the Residue Theorem 70
1. The winding number 70
2. The residue theorem 75
3. Applications of the residue theorem 81

Notes on Chapter 3 87
References: Chapter 3 87

Chapter 4. Picard's Theorem 89

Notes on Chapter 4 97
References: Chapter 4 98

Chapter 5. The Inhomogeneous Cauchy-Riemann Equation and Runge's Theorem 100
1. Partitions of unity 100
2. The equation $\frac{\partial u}{\partial \bar{z}} = \phi$ 102
3. Runge's theorem 106
4. The homology form of Cauchy's theorem 115

Notes on Chapter 5 117
References: Chapter 5 118

Chapter 6. Applications of Runge's Theorem 119
1. The Mittag-Leffler theorem 119
2. The cohomology form of Cauchy's theorem 123
3. The theorem of Weierstrass 126
4. Ideals in $\mathscr{H}(\Omega)$ 131

Notes on Chapter 6 141
References: Chapter 6 142

Chapter 7. The Riemann Mapping Theorem and Simple Connectedness in the Plane 144
1. Analytic automorphisms of the disc and of the annulus 144
2. The Riemann mapping theorem 148
3. Simply connected plane domains 150

Notes on Chapter 7 153
References: Chapter 7 154

Chapter 8. Functions of Several Complex Variables 156

Notes on Chapter 8 164
References: Chapter 8 165

Chapter 9. Compact Riemann Surfaces 166
1. Definitions and basic theorems 166
2. Meromorphic functions 171
3. The cohomology group $H^1(\mathfrak{U},\mathcal{O})$ 173
4. A theorem from functional analysis 176
5. The finiteness theorem 183
6. Meromorphic functions on a compact Riemann surface 185

Notes on Chapter 9 190
References: Chapter 9 191

Chapter 10. The Corona Theorem 193
1. The Poisson Integral and the theorem of F. and M. Riesz 194
2. The corona theorem 204

Notes on Chapter 10 214
References: Chapter 10 215

Chapter 11. Subharmonic Functions and the Dirichlet Problem 216
1. Semi-continuous functions 216
2. Harmonic functions and Harnack's principle 219
3. Convex functions 223
4. Subharmonic functions: Definition and basic properties 227
5. Subharmonic functions: Further properties and application to convexity theorems 236
6. Harmonic and subharmonic functions on Riemann surfaces 246
7. The Dirichlet problem 246
8. The Radó-Cartan theorem 255

Notes on Chapter 11 260
References: Chapter 11 261

Appendix 263

Subject Index 264