

## Contents

Chapter I. Boundary Value Theory for Harmonic and Holomorphic Functions in the Unit Disk . . . . .	1
1. Harmonic Functions . . . . .	1
2. Pointwise Convergence: The Fatou Theorem and its Converse. . . . .	6
3. Holomorphic Functions. . . . .	12
4. The Function Classes $\text{Hol}^\#(D)$ and $H^\#(D)$ . . . . .	16
Notes. . . . .	21
Chapter II. Function Algebras: The Bounded-Measurable Situation. . . . .	22
1. Szegö Functional and Fundamental Lemma . . . . .	22
2. Measure Theory: Prebands and Bands . . . . .	26
3. The abstract F. and M. Riesz Theorem . . . . .	31
4. Gleason Parts. . . . .	34
5. The abstract Szegö-Kolmogorov-Krein Theorem. . . . .	36
Notes. . . . .	42
Chapter III. Function Algebras: The Compact-Continuous Situation . . . . .	44
1. Representative Measures and Jensen Measures. . . . .	44
2. Return to the abstract F. and M. Riesz Theorem . . . . .	47
3. The Gleason and Harnack Metrics. . . . .	48
4. Comparison of the two Gleason Part Decompositions. . . . .	54
Notes. . . . .	58
Chapter IV. The Abstract Hardy Algebra Situation . . . . .	59
1. Basic Notions and Connections with the Function Algebra Situation. . . . .	60
2. The Functional $\alpha$ . . . . .	66
3. The Function Classes $H^\#$ and $L^\#$ . . . . .	69
4. The Szegö Situation. . . . .	76
Notes. . . . .	79
Chapter V. Elements of Abstract Hardy Algebra Theory . . . . .	81
1. The Moduli of the invertible Elements of $H^\#$ . . . . .	81
2. Substitution into entire Functions . . . . .	84

3. Substitution into Functions of Class $\text{Hol}^{\#}(D)$ . . . . .	85
4. The Function Class $H^{+}$ . . . . .	91
5. Weak- $L^1$ Properties of the Functions in $H^{+}$ . . . . .	97
6. Value Carrier and Lumer Spectrum . . . . .	102
Notes. . . . .	106
Chapter VI. The Abstract Conjugation . . . . .	108
1. A Representation Theorem . . . . .	110
2. Definition of the abstract Conjugation . . . . .	111
3. Characterization of $E$ with the means of $M$ . . . . .	115
4. The basic Approximation Theorem. . . . .	119
5. The Marcel Riesz and Kolmogorov Estimations. . . . .	126
6. Special Situations . . . . .	138
7. Return to the Marcel Riesz and Kolmogorov Estimations. . . . .	144
Notes. . . . .	146
Chapter VII. Analytic Disks and Isomorphisms with the Unit Disk Situation . . . . .	149
1. The Invariant Subspace Theorem . . . . .	149
2. The Maximality Theorem . . . . .	151
3. The Analytic Disk Theorem. . . . .	155
4. The Isomorphism Theorem. . . . .	160
5. Complements on the simple Invariance of $H_{\varphi}$ . . . . .	165
6. A Class of Examples. . . . .	167
Notes. . . . .	170
Chapter VIII. Weak Compactness of $M$ . . . . .	172
1. The Decomposition Theorem of Hewitt-Yosida . . . . .	172
2. Strict Convergence . . . . .	175
3. Characterization Theorem and Main Result . . . . .	177
Notes. . . . .	180
Chapter IX. Logmodular Densities and Small Extensions. . . . .	181
1. Logmodular Densities . . . . .	181
2. The Closed Subgroup Lemma. . . . .	186
3. Small Extensions . . . . .	190
Notes. . . . .	197

Chapter X. Function Algebras on Compact Planar Sets. . . . .	198
1. Consequences of the abstract Hardy Algebra Theory. . . . .	199
2. The Cauchy Transformation of Measures. . . . .	204
3. Basic Facts on $P(K) \subset R(K) \subset A(K)$ . . . . .	209
4. On the annihilating and the representing Measures for $R(K)$ and $A(K)$ . . . . .	214
5. On the Gleason Parts for $R(K)$ and $A(K)$ . . . . .	218
6. The Logarithmic Transformation of Measures and the Logarithmic Capacity of Planar Sets. . . . .	221
7. The Walsh Theorem. . . . .	227
8. Application to the Problem of Rational Approximation . . . .	231
Notes. . . . .	234
Appendix . . . . .	236
1. Linear Functionals and the Hahn-Banach Theorem . . . . .	236
2. Measure Theory . . . . .	239
3. The Cauchy Formula via the Divergence Theorem. . . . .	241
Notes. . . . .	244
References . . . . .	245
Notation Index . . . . .	255
Subject Index. . . . .	257