

# Contents

## Chapter 1

### Geometric Function Theory

1.1. Basic Principles	1
1.2. Local Mapping Properties	5
1.3. Normal Families	7
1.4. Extremal Problems	10
1.5. The Riemann Mapping Theorem	11
1.6. Analytic Continuation	12
1.7. Harmonic and Subharmonic Functions	15
1.8. Green's Functions	19
1.9. Positive Harmonic Functions	21
Exercises	24

## Chapter 2

### Elementary Theory of Univalent Functions

2.1. Introduction	26
2.2. The Area Theorem	29
2.3. Growth and Distortion Theorems	32
2.4. Coefficient Estimates	36
2.5. Convex and Starlike Functions	40
2.6. Close-to-Convex Functions	46
2.7. Spirallike Functions	52
2.8. Typically Real Functions	55
2.9. A Primitive Variational Method	58
2.10. Growth of Integral Means	60
2.11. Odd Univalent Functions	64
2.12. Asymptotic Bieberbach Conjecture	66
Notes	69
Exercises	70

## Chapter 3

### Parametric Representation of Slit Mappings

3.1. Carathéodory Convergence Theorem	76
3.2. Density of Slit Mappings	80
3.3. Loewner's Differential Equation	82
3.4. Univalence of Solutions	87

3.5. The Third Coefficient	93
3.6. Radius of Starlikeness	95
3.7. The Rotation Theorem	98
3.8. Coefficients of Odd Functions	103
3.9. An Elementary Counterexample	107
3.10. Robertson's Conjecture	110
3.11. Successive Coefficients	113
Exercises	115
Chapter 4	
Generalizations of the Area Principle	118
4.1. Faber Polynomials	118
4.2. Polynomial Area Theorem	120
4.3. The Grunsky Inequalities	122
4.4. Inequalities of Goluzin and Lebedev	125
4.5. Unitary Matrices	128
4.6. The Fourth Coefficient	131
4.7. Coefficient Problem in the Class $\Sigma$	134
Notes	139
Exercises	140
Chapter 5	
Exponentiation of the Grunsky Inequalities	142
5.1. Exponentiation of Power Series	142
5.2. Reformulation of the Grunsky Inequalities	146
5.3. Estimation of the $n$ th Coefficient	149
5.4. Logarithmic Coefficients	151
5.5. Radial Growth	157
5.6. Bazilevich's Theorem	159
5.7. Hayman's Regularity Theorem	162
5.8. Proof of Milin's Tauberian Theorem	168
5.9. Successive Coefficients	172
5.10. Successive Coefficients of Starlike Functions	177
5.11. Exponentiation of the Goluzin Inequalities	180
5.12. FitzGerald's Theorem	183
Exercises	187
Chapter 6	
Subordination	190
6.1. Basic Principles	190
6.2. Coefficient Inequalities	192
6.3. Sharpened Forms of the Schwarz Lemma	197
6.4. Majorization	202
6.5. Univalent Subordinate Functions	207
Exercises	212

## Chapter 7

<b>Integral Means</b>	214
7.1. Baernstein's Theorem	214
7.2. The Star-Function	216
7.3. Proof of Baernstein's Theorem	219
7.4. Subharmonic Property of the Star-Function	225
7.5. Integral Means of Derivatives	229
Exercises	232

## Chapter 8

<b>Some Special Topics</b>	234
8.1. Bounded Univalent Functions	234
8.2. Sections of Univalent Functions	243
8.3. Convolutions of Convex Functions	246
8.4. Coefficient Multipliers	254
8.5. Criteria for Univalence	258
8.6. Additional Topics	265
1. Bieberbach-Eilenberg Functions	265
2. Univalent Polynomials	267
3. Functions of Bounded Boundary Rotation	269
Exercises	271

## Chapter 9

<b>General Extremal Problems</b>	275
9.1. Functionals on Linear Spaces	275
9.2. Representation of Linear Functionals	278
9.3. Extreme Points and Support Points	280
9.4. Properties of Extremal Functions	283
9.5. Extreme Points of $S$	286
9.6. Extreme Points of $\Sigma$	288
Exercises	290

## Chapter 10

<b>Boundary Variation</b>	292
10.1. Preliminary Remarks	292
10.2. Conformal Radius	293
10.3. Schiffer's Theorem	295
10.4. Local Structure of Trajectories	302
10.5. Application to Extremal Problems	304
10.6. Support Points of $S$	306
10.7. Point-Evaluation Functionals	314
10.8. The Coefficient Problem	318
10.9. Region of Values of $\log f(\zeta)/\zeta$	323
10.10. Multiply Connected Domains	326
10.11. Other Variational Methods	328
Exercises	330

Chapter 11	
<b>Coefficient Regions</b>	334
11.1. Elementary Properties	334
11.2. Boundary Points	338
11.3. Canonical Differential Equation	343
11.4. Algebraic Functions	346
Exercises	352
<b>Suggestions for Further Reading</b>	355
<b>Bibliography</b>	357
<b>List of Symbols</b>	377
<b>Index</b>	379