## Contents

Prei	ace	XII
Intro	oduction	xiv
	Chapter 1	
	Systems of implicit functions and the classical branching theory	
§1.	The implicit function problem	1
§2.	The one-dimensional branching case and Newton's diagram 2.1. Newton's diagram (10). 2.2. Properties of the solutions (12). 2.3. Examples (16). 2.4. Investigation of the branching equations. The case of simple roots of the defining equation (18). 2.5. The case of multiple roots of the defining equation (22). 2.6. Real solutions (24). 2.7. Special configurations of Newton's diagram (26).	9
	Chapter 2	
	Investigation of the branching equation in the many-dimensional case	
§3.	Transformation of the branching equation	34
§4.	Topics in divisibility theory	41

§ 5.	The two-dimensional branching case	
	5.1. General investigation of the problem (53). 5.2. Branching indices in the two-dimensional case (56). 5.3. Special cases (59).	52
§6.	The many-dimensional branching case	70
	Chapter 3	
	The branching equation for non-linear integral and integro-differential equations	
§7.	The Lyapunov-Schmidt integral equations	80
§8.	The general Lyapunov-Schmidt integral equation 8.1. The regular case (100). 8.2. The regular case with many arguments (102). 8.3. Schmidt's lemma (103). 8.4. The one-dimensional branching case (104). 8.5. The many-dimensional branching case (108). 8.6. Possible generalizations (111).	99
§9.	Lyapunov-Schmidt systems of equations and certain integro- differential equations	112

	Chapter 4	
	General integral equation and the coefficients of the branching equation	
§10.	The general integral equation	134
§11.	The coefficients of the branching equations	167
	Chapter 5	
	Characterization and construction of solutions of non-linear equations	
<b>§12</b> .	Characterization of solutions of non-linear integral equations	201
§13.	Construction of solutions to non-linear integral equations	222

sional branching case (224). 13.3. The two-dimensional branching case (228). 13.4. The Nekrasov equation (233). 13.5. The branching equation

for Nekrasov's equation (235).

§14	14.1. Formulation of the problem (242). 14.2. Reduction to the problem of small solutions (242). 14.3. Singular solutions in the regular case (243). 14.4. Investigation of the auxiliary equation (245). 14.5. Branching of the solutions of the fundamental equation (247). 14.6. Singular solutions in Lebesgue spaces (249).	242
	Chapter 6	
	Branching of periodic solutions of differential equations	
§15.	Periodic solutions of non-autonomous systems	251
§16.	Periodic solutions of quasi-linear systems	258
§17.	Periodic solutions of autonomous systems  17.1. The Poincaré problem for autonomous systems (263). 17.2. The branching equation (264). 17.3. The main results (266). 17.4. The method of undetermined coefficients (267). 17.5. Autonomous systems with one degree of freedom (269).	263
§18.	Examples	272
§19.	Additional problems involving periodic solutions 19.1. Singular periodic solutions of non-autonomous systems (283). 19.2. Branching of periodic solutions in Banach spaces (285).	283
§20.	On the stability of periodic solutions dependent on a small parameter	290

## Chapter 7

## Non-linear equations in Banach spaces

§21.	Some topics in the theory of linear operators in Banach spaces	306
§22.	Power operators, Taylor series, the implicit operator theorems	314
§23.	The Lyapunov-Schmidt branching equation	323
§24.	Investigation of the one-dimensional branching case 24.1. Calculation of the leading coefficients (331). 24.2. The degenerate case (334). 24.3. The quasi-regular case (Problem A) (335). 24.4. Problem B (the non-degenerate case) (228). 24.5. The real case (339). 24.6. The branching of solutions of equations with sufficiently smooth operators (340). 24.7. The case of a functional parameter. Series in powers of homogeneous operators (341). 24.8. The case of two numerical parameters (345).	331
§25.	The many-dimensional branching case	347

## Chapter 8 Branching of the solutions of non-linear equations in the singular case 354 26.1. Noether operators (354). 26.2. Decomposition of spaces into a direct sum of subspaces. Restriction of an operator (355). 26.3. Atkinson's theorem. The relation to adjoint operators (356). 360 27.1. Formulation of the problem (360). 27.2. The case n > 0, m = 0(360). 27.3. The case n = 0, m > 0 (362). 27.4. The fundamental case (362). 27.5. Branching of solutions of an equation with an unbounded operator (363). §28. Branching of solutions of non-linear singular integral equations . . . . . 364 28.1. Linear singular integral operators with a Cauchy kernel in Hölder spaces (364). 28.2. Non-linear singular integral equations with a Cauchy kernel in Hölder spaces (366). 28.3. The analytic case (368). 28.4. Nonlinear singular integral equations with a Hilbert kernel in Lebesgue spaces (370).§29. Branching of solutions of boundary-value problems for non-375 29.1. Boundary-value problems for second-order elliptic equations in Hölder spaces (375). 29.2. Boundary-value problems in the plane for k-th order elliptic systems in a Hölder space (380). 29.3. Boundary-value problems for elliptic equations in the space of summable functions (382). Chapter 9 Selected problems in perturbation theory § 30. Jordan chains and systems of Fredholm operators . . . . . 387 30.1. A-Jordan chains for n = 1 (387). 30.2. A-Jordan chains and sets where n > 1 (389). 30.3. Conditions for the completeness of an A-Jordan set (391). 30.4. Example (395).

31.1. The case n = 1 (396). 31.2. The case n > 1 (399). 31.3. The method

396

§31. Perturbation of a linear equation by a small linear term

of undetermined coefficients (400).

§32.	Branching of eigenvalues and eigenelements of Fredholm operators	404
	32.1. Derivation of the branching equation (404). 32.2. The branching equation in the analytical case (406). 32.3. The degenerate and the non-degenerate case (407). 32.4. The one-dimensional case (409). 32.5. The method of undetermined coefficients (413). 32.6. The many-dimensional case (418).	,,,,
§33.	Singular solutions of non-linear equations	424
	Chapter 10	
	Applied problems	
§ 34.	Small bending deformation of a straight rod under constant load	450
§35.	The theory of small deflections in elastic plates	454
§36.	Oscillations of a satellite in the plane of an elliptic orbit	465
§37.	Standing waves	469
Bibli	ography	478
Subj	ect index	486