

Contents

Introduction	1
Chapter I • Dirichlet Finite Analytic Functions	
§ 1. Arbitrary Surfaces	10
1. Modular Test	11
1A. Modulus 11 — 1B. Geometric Meaning 11 — 1C. Generalization 12 — 1D. Modular Test 14 — 1E. Example 15 — 1F. Relative Class SO_{AD} 16 — 1G. Classes O_{A_0D} and O_{A^0D} 17 — 1H. Test for O_{A_0D} and O_{A^0D} 18	
2. Conformal Metric Test	18
2A. Conformal Metric 18 — 2B. Conformal Metric Test 19 — *2C. Fundamental Polygons 20 — *2D. Euclidean Metric Test 21	
3. Regular Chain Test	22
3A. Regular Chains 22 — 3B. Regular Chain Test 22 — *3C. Second Proof 24 — 3D. Comments on Regular Chains 25 — 3E. Concluding Remarks 25	
§ 2. Plane Regions	25
4. Convergent Modular Products	26
4A. Estimate for Modulus 26 — 4B. Bisecting the Annulus 27 — 4C. Second Proof 28 — 4D. Convergent Modular Products 29	
5. Relative Width Test	29
5A. Relative Width 29 — 5B. Relative Width Test 30 — 5C. Square Net Test 31	
6. Generalized Cantor Sets	32
6A. Vanishing Linear Measure 32 — 6B. Zero Area 34 — 6C. Regions of Area ε 34	
7. Extremal Functions and Conformal Mappings	35
7A. Principal Functions 35 — 7B. Proof 36 — 7C. Operators L_0 and L_1 38 — 7D. Functions with Singularities 39 — 7E. Conformal Mappings 40 — 7F. Principal Functions P_j^θ 41 — 7G. Univalence of P_0^θ 42 — 7H. An Extremal Property of P_0^θ 42 — 7I. Horizontal Slits 43 — 7J. Mappings P_0 and P_1 43 — 7K. Mapping $P_0 + P_1$ 43	

8. Characterization of O_{AD} -Regions	44
8A. The Analytic Span 44 — 8B. Regular Functions 45 — 8C. Characterizations 46 — 8D. Removable Sets 47 — 8E. Surfaces of Finite Genus 48 — 8F. Closed Extensions 49 — 8G. Closed Extensions (continued) 49	
9. Class ABD on Surfaces of Finite Genus	50
9A. $O_{AD} = O_{ABD}$ for Finite Genus 50 — *9B. Finite ABD -Interpolation 51 — *9C. Finite AD -Interpolation with Minimum Norm 52	
10. Essential Extendability	53
10A. Finite Genus 53 — 10B. Infinite Genus 53 — 10C. Boundary Property 54 — 10D. Relations $O_{A^0D} < O_{A_0D}$ and $O_{A^0D} < O_{AD}$ 55	
11. Koebe's Circular Mappings	55
11A. Koebe's Principle 55 — 11B. Exhaustion by Noncompact Regions 55 — 11C. Regular Chain Sets 55 — 11D. Test for Circular Mappings 56 — 11E. An Application 57	
§ 3. Covering Surfaces of the Sphere	58
12. Finite Sets of Projections of Branch Points	58
12A. Surface Elements 58 — 12B. Polyhedral Representation 59 — 12C. O_{AD} -Test 59 — 12D. Construction of an Exhaustion 60 — 12E. Disk Chains Covering α 60 — 12F. Disk Chains Covering β 61 — 12G. Enumeration 61	
*13. Application to Planar Surfaces	61
13A. Line Complexes 61 — 13B. O_{AD} -Test 62 — 13C. Edge Sequences with Bounded Vertex Numbers 63 — 13D. Finite Sets of Branch Points 63 — 13E. Periodic Ends 63	
*14. Nonplanar Surfaces	64
14A. Strip Complexes 64 — 14B. Example 64	
*15. Punctured Surfaces	65
15A. Polyhedral Representation of a Punctured Surface 65 — 15B. O_{AD} -Test 66 — 15C. Chains Relative to $\{\sigma_n\}$ 66 — 15D. Chains Relative to $\{h_n\}$ 67 — 15E. Construction of Exhaustion 68 — 15F. Evaluation of D_R 68	
*16. Finite Sets of Sheets	69
16A. Regular and Singular Projections 69 — 16B. O_{AD} -Test 69	
§ 4. Covering Surfaces of Riemann Surfaces	70
17. Preliminaries	70
17A. Problem 70 — 17B. Covering Surfaces of the Torus 70 — 17C. Pair of Cuts 71 — 17D. Covering Surfaces of the Double Torus 71 — 17E. Covering Surface $\tilde{R}(a_1)$ 71 — 17F. Covering Surface	

$\tilde{R}(a_1 + b_1)$ 72 — **17 G.** Covering Surface $\tilde{R}(a_1, a_2)$ 72 — **17 H.** Covering Surface $\tilde{R}(a_1 + b_1, a_2)$ 72 — **17 I.** Covering Surface $\tilde{R}(a_1 + b_1, a_2 + b_2)$ 72

18. Covering Surfaces of Closed Surfaces 72
18 A. Definitions 72 — **18 B.** Main Theorem 73 — **18 C.** Schottky Point Sets 74

***19.** Covering Surfaces of Open Surfaces 74
19 A. Covering Surfaces Associated with a Set of Cycles 74 — **19 B.** Abelian and Schottky Covering Surfaces 75 — **19 C.** Covering by Regular Chains 75 — **19 D.** O_{AD} -Test 76 — **19 E.** Finite Genus 77 — **19 F.** Transcendental Hyperelliptic Surfaces 77 — **19 G.** Strip Complexes 78

Chapter II · Other Classes of Analytic Functions

§ 1. Inclusion Relations 79

1. Basic Inclusions 80
1 A. Plane Regions 80 — **1 B.** Arbitrary Surfaces 80

2. The Class O_{AB} 80
2 A. Conformal Metric Test 80 — **2 B.** Modular Test 83 — **2 C.** Relative Classes 84 — **2 D.** Plane Regions 84 — **2 E.** Hausdorff Measure 85 — **2 F.** Newtonian Potential 86 — **2 G.** Newtonian Capacity 86 — **2 H.** The Class N 88 — **2 I.** Associated Measure 88 — **2 J.** The Proof of $N \subset M_{1+\epsilon}$ 89 — **2 K.** O_{AB} -Test and Linear Measure 90 — **2 L.** Test for Linear Measure Zero 91 — **2 M.** Boundary Property 92

****3.** Covering Surfaces of Closed Surfaces 93
3 A. Commutative Covering Surfaces 93 — **3 B.** Generators of $G(\tilde{R})$ 93 — **3 C.** Construction of \tilde{R} 95 — **3 D.** Structures of \tilde{R} and $G(\tilde{R})$ 95 — **3 E.** Standard Exhaustion $\{\tilde{R}_n\}$ of \tilde{R} 96 — **3 F.** Main Theorem 97 — **3 G.** Estimation of Length of $\partial\tilde{R}_n$ 98 — **3 H.** Length of Components of $\partial\tilde{R}_n$ 99 — **3 I.** Membership in O_{AB} 100

§ 2. Plane Regions and Conformal Invariants 101

4. The Invariant M_F 102
4 A. Weak and Strong Monotonic Properties 102 — **4 B.** Compact Function Classes 103 — **4 C.** Special Classes 103

5. Invariants M_{AB} and M_{AE} 104
5 A. Equality of Invariants 104 — **5 B.** Vanishing of $M_{AB} = M_{AE}$ 105 — **5 C.** Meromorphic Functions 106 — **5 D.** Painlevé Null Sets 106

6. Invariants M_{AD} and M_{SE} 106
6 A. Equality of Invariants 106 — **6 B.** Inequality $M_{AD} \leq M_{AB}$ 107

7. The Invariant $M_{AD}(z_1, z_2, R)$	107
7 A. A Characterization 107 — 7 B. Circular and Radial Slit Mappings 107 — 7 C. Evaluation of $M_{AD}(z_1, z_2, R)$ 108	
8. Invariants M_{SD} and M_{SB}	109
8 A. Extremal Length 109 — 8 B. Elementary Properties 110 — 8 C. Perimeter of a Set 112 — 8 D. Perimeter of a Point 113 — 8 E. $M_{SB} = \max \mu$ for a Regular Region 113 — 8 F. $M_{SD} = M_{SB}$ for a Regular Region 114 — 8 G. The General Case 115	
9. O_{AD} -Regions and Extremal Distances	115
9 A. Extremal Distance 115 — 9 B. General Form 117 — 9 C. Pro- jections 117	
10. Linear Sets	118
10 A. Linear Measure and M_{AB} 118 — 10 B. Invariants M_{SB} and M_{AD} 119 — 10 C. Sets of Capacity Zero 120 — 10 D. Sets on a Circle 120 — 10 E. Circular Sets with O_{AD} -Complements 121	
11. Counterexamples	121
11 A. General Relations 121 — 11 B. M_1 and O_{SB} 122 — 11 C. M_2 and O_{SB} 124 — 11 D. M_1 and O_{AD} 124 — 11 E. Strict Inclusion $M_1 < O_{AB}$ 125 — 11 F. Positive Length 125 — 11 G. Analytic Capacity 126 — 11 H. Cauchy Potentials 127 — 11 I. Perturbation 129 — 11 J. An Estimate 130 — 11 K. Completion of the Proof 130	
§ 3. <i>K</i> -Functions	131
12. Inclusion Relations and Tests	132
12 A. Basic Inclusions 132 — 12 B. Inclusions for H 133 — 12 C. O_{KD} -Test 133	
13. Characterization of O_{KD}	134
13 A. Spaces \mathcal{D}^∞ and \mathcal{D} 134 — 13 B. Spaces \mathcal{D}_K^∞ and \mathcal{D}_K 134 — 13 C. Characterization of O_{KD} 134	
14. Quasiconformal Mappings and Boundary Properties	135
14 A. Quasiconformal Mappings as Dirichlet Mappings 135 — 14 B. Quasiconformal Invariance of O_{KD} 135 — 14 C. Quasiconformal Noninvariance of O_{KB} 135 — 14 D. Boundary Properties 136	
15. Surfaces of Finite Genus	137
15 A. Identities for A and K 137 — 15 B. Quasiconformal Invariance of O_{AD} for Finite Genus 137 — 15 C. Surfaces with Holes 137 — 15 D. Strict Inclusions 137 — 15 E. The Class O_{KP} 138	
*16. The Riemann-Roch Theorem	138
16 A. Divisors 138 — 16 B. Relations 139 — 16 C. Relations as Principal Parts 139 — 16 D. An Auxiliary Formula 140 — 16 E. Solu- tions of $L_f(x) = 0$ 140 — 16 F. Characterization of Principal Parts 141 — 16 G. Neumann's Function 141 — 16 H. Sufficiency 141 — 16 I. Generalized Riemann-Roch Theorem 142 — 16 J. Classical Case 143	

Chapter III · Dirichlet Finite Harmonic Functions

§ 1. Royden's Compactification	145
1. Royden's Algebra	147
1 A. Tonelli Functions 147 — 1 B. Definition of Royden's Algebra 148 — 1 C. Completeness 148 — 1 D. Approximation 150 — 1 E. Green's Formula 151 — 1 F. Dirichlet's Principle 153 — 1 G. Potential Subalgebra 153 — 1 H. Ideals 154	
2. Royden's Compactification	154
2 A. Definition of Royden's Compactification 154 — 2 B. Characters 155 — 2 C. Urysohn's Property 155 — 2 D. Royden's Boundary 156 — 2 E. Harmonic Boundary 156 — 2 F. Parabolic Surfaces 157 — 2 G. Maximum Principle I 159 — 2 H. Maximum Principle II 159 — 2 I. Maximum Principle III 160 — 2 J. Duality 160	
3. Orthogonal Projection	161
3 A. Quasi-Dirichlet Finiteness 161 — 3 B. Orthogonal Decomposition 162 — 3 C. Reformulation 164 — 3 D. Orthogonal Projection 165 — 3 E. HD -Minimal Functions 165 — 3 F. A Characterization of O_{HD} 166 — 3 G. Space HD of Finite Dimension 166 — 3 H. Evans' Superharmonic Function 167 — 3 I. Maximum Principle IV 168 — 3 J. Dirichlet Integral of the Harmonic Measure 168	
§ 2. Dirichlet's Problem	171
4. Harmonic Measure and Kernel	171
4 A. Harmonic Measure on Γ 171 — 4 B. Harmonic Kernel 173 — 4 C. Harnack's Function 174 — 4 D. Harmonicity of $P(z, p)$ 175 — 4 E. Integral Representation 176 — 4 F. Vector Lattice HD 177 — 4 G. The Identity $O_{HBD} = O_{HD}$ 178 — 4 H. Strict Inclusion $O_{HB} < O_{HD}$ 178 — 4 I. The Class \tilde{HD} 181 — 4 J. Upper Semicontinuous Functions on Δ 182 — 4 K. Boundary Function 183 — 4 L. Semivector Lattice \tilde{HD} 184 — 4 M. Characterization of \tilde{HD} -Minimality 186 — 4 N. Characterization of $U_{\tilde{HD}}$ 187	
5. Perron's Method	187
5 A. Perron's Family 187 — 5 B. Compactification of Subregions 189 — 5 C. Coincidence of Boundary Points 190 — 5 D. Correspondence of Harmonic Measures I 191 — 5 E. Correspondence of Harmonic Measures II 192 — 5 F. Surfaces of Almost Finite Genus 193 — 5 G. $O_G = O_{HD}$ for Almost Finite Genus 194 — 5 H. Boundary Theorem of Riesz-Lusin-Privaloff Type 194 — 5 I. The Inclusion $U_{\tilde{HD}} < O_{AD}$ 196 — 5 J. Examples of O_{HD}^n -Surfaces 197	
6. Green's Lines	197
6 A. Polar Coordinates 197 — 6 B. Space of Green's Lines 199 — 6 C. Ends of Green's Lines 201 — 6 D. Radial Limits 203 — 6 E. Lat-	

tice of Radial Limits	204	—	6 F. Gauss' Property of Radial Limits	205
6 G. Functions with Radial Limits Zero	205	—	6 H. Harmonic and Green's Measures	205
—		—	6 I. Boundary Theorem of Riesz Type	206
6 J. Blocks	206	—	6 K. Another Characterization of $U_{\tilde{H}D}$	208
6 L. Second Proof of $U_{\tilde{H}D} \subset O_{AD}$	209			

§ 3. Invariance under Deformation 209

7. Algebraic Structure 210

7 A. Quasiconformal Mappings	210	—	7 B. Annular Functions	210
7 C. Algebraic Characterization	211	—	7 D. Analytic Properties	212
—		—	7 E. Existence of the Isomorphism	212
7 F. Existence of a Topological Map	213	—	7 G. Quasiconformality	214
7 H. Conformal Equivalence	216			

8. Topological Structure 216

8 A. A -Sets	216	—	8 B. Royden's Mapping	216
—		—	8 C. Topological Characterization	217
8 D. Topological Extension	218	—	8 E. Restriction	219
8 F. Boundary Behavior	219	—	8 G. Boundary Behavior (continued)	220
—		—	8 H. Invariance of $O_G, O_{HD}, O_{\tilde{H}D}^n$, and U_{HD}	221
8 I. Boundary Property	221			

Chapter IV · Other Classes of Harmonic Functions

§ 1. Wiener's Compactification 222

1. Wiener's Algebra 223

1 A. Harmonizable Functions	223	—	1 B. Definition of Wiener's Algebra	223
—		—	1 C. Potential Subalgebra	224
1 D. Properties of $\mathbb{N}(R)$	226	—	1 E. Completeness	226
—		—	1 F. Lattice	227
1 G. The Inclusion $M\mathbb{I}(R) \subset \mathbb{N}(R)$	227			

2. Wiener's Compactification 228

2 A. Definition of Wiener's Compactification	228	—	2 B. Characters	228
—		—	2 C. The Identity $\mathbb{N}(R) = B(R_{\mathbb{N}}^*)$	229
2 D. Čech Compactification	229	—	2 E. Wiener's Boundary	229
—		—	2 F. The Fiber Space $(R_{\mathbb{N}}^*, R_{M\mathbb{I}}^*, \rho)$	230
2 G. Remarks on $\Gamma_{\mathbb{N}}$ and $\Delta_{\mathbb{N}}$	231	—	2 H. The Class \mathscr{W} for $R \notin O_G$	231
2 I. The Class \mathscr{W} for $R \in O_G$	232			

3. Harmonic Projection 233

3 A. Positive Harmonic Functions	233	—	3 B. Bounded Harmonic Functions	234
—		—	3 C. Strict Inclusion $O_{HP} < O_{HB}$	235
3 D. Maximum Principle V	235	—	3 E. Harmonic Decomposition	236
—		—	3 F. The Space $\mathscr{W}_A(R)$	236
3 G. The Space $\mathscr{W}_{A \cup K}(R)$	237	—	3 H. Harmonic Projection	239
—		—	3 I. Evans' Superharmonic Function	239
3 J. Maximum Principle VI	240	—	3 K. The Class U_{HB}	240
3 L. Relative Classes SO_{HB} and SO_{HD}	241	—	3 M. Two Region Test	242
3 N. Strict Inclusions	243			

§ 2. Dirichlet's Problem	244
4. Harmonic Measure and Kernel	244
4 A. Harmonic Measure on $\Gamma_{\mathbb{N}}$ 244 — 4 B. Harmonic Kernel 245 —	
4 C. Stonean Space $\Delta_{\mathbb{N}}$ 245 — 4 D. Integral Representation 246 —	
4 E. Operator B' 248 — 4 F. Evans' Harmonic Function for a Set in $\Delta_{\mathbb{N}}$ 248	
5. Perron's Method	249
5 A. Perron's Family 249 — 5 B. Measure Correspondence between $\Gamma_{\mathbb{N}}$ and Γ_{M_1} 250 —	
5 C. Compactifications of Subsurfaces 250 — 5 D. Stoilow's Compactification $R_{\mathbb{N}}^*$ 250 —	
5 E. Harmonic Measure on Γ_S 252 — 5 F. Test for Quasiboundedness 252	
6. Φ -Bounded Harmonic Functions	254
6 A. Φ -Boundedness 254 — 6 B. Determination of $O_{H\Phi}$ 255 —	
6 C. Harmonic Measures on the Disk 257 — 6 D. Convergence on the Boundary 259 —	
6 E. First Example 261 — 6 F. Second Example 264	
6 G. The Inclusion $H\Phi \subset HP'$ 264 — 6 H. The Inclusion $H\Phi \cap HP' \subset HB'$ 265 —	
6 I. The Relative Class $SO_{H\Phi}$ 266 — 6 J. The Class $SO_{H\Phi}$ for $\bar{d}(\Phi) < \infty$ 266 —	
6 K. Increasing Convex Φ 267 — 6 L. Φ -Mean Boundedness 268 —	
6 M. A Relation to HD 268	
§ 3. Lindelöfian Meromorphic Functions	269
7. Inclusion Relations	269
7 A. Boundary Theorem of Riesz-Lusin-Privaloff Type 269 — 7 B. Lindelöfian Meromorphic Functions 270 —	
7 C. The Inclusion $O_G \subset O_{MB^*}$ 270 — 7 D. The Decomposition Theorem 271 —	
7 E. Continuity on $R_{\mathbb{N}}^*$ 271 — 7 F. Properties of $B'u(z; f-a)$ 271 —	
7 G. The Inclusion $U_{HB} \subset O_{MB^*}$ 272	
8. Covering Properties	272
8 A. Capacity of a Plane Set 272 — 8 B. Capacity of $f(\Delta_{\mathbb{N}})$ 273 —	
8 C. Meromorphic Functions on O_{MB^*} -Surfaces 273 — 8 D. Parabolic Ends 274 —	
8 E. Bounded Valence 276	
9. Examples Concerning O_{MB^*}	276
9 A. Lindelöfian Analytic Functions 276 — 9 B. Picard's End 278 —	
9 C. General O_{MB^*} 280 — 9 D. O_{MB^*} for Finite Genus 280	
§ 4. Invariance under Deformation	281
10. Wiener's Structure	281
10 A. Wiener's Mapping 281 — 10 B. Algebraic Structure 281 —	
10 C. Topological Structure 282	
11. Boundary Behavior	282
11 A. Invariance of Harmonic Boundary 282 — 11 B. Absolute Continuity 284 —	
11 C. Invariant Classes 284 — 11 D. Boundary Property 285	

Chapter V • Functions with Logarithmic Singularities

§ 1. Capacity Functions	286
1. Capacity of the Boundary	287
1A. An Inequality 287 — 1B. Minimum Property 287 — 1C. Convergence Proof 288 — 1D. Deviation Formula 289 — 1E. Capacity c_β 290	
2. The Class of Functions W	290
2A. Minimum of $D(W)$ 290 — 2B. Schwarz's Lemma 291 — 2C. Minimax Property 292	
3. Capacity of an Ideal Boundary Point.	292
3A. Minimum Property 292 — 3B. Capacity c_γ 294 — 3C. The Class $\{V\}$ 294 — 3D. Subboundaries 295	
4. Surface Classes C_β and C_γ	295
4A. Weak Boundaries 295 — 4B. Absolutely Disconnected Boundaries 295	
5. Strong and Weak Components	296
5A. Boundary Components under Conformal Mappings 296 — 5B. Weak Boundary Components 297 — 5C. Tests 297 — 5D. Capacity c^γ 297 — 5E. Strong Boundary Components 298 — 5F. Minimum Property 298 — 5G. Reduction Theorem 298	
6. Univalent Functions	299
6A. The Class \mathcal{F} 299 — 6B. Maximal Disks 301 — 6C. Minimal Disks 301 — 6D. Rigid Disks 302	
§ 2. Parabolic and Hyperbolic Surfaces	303
7. Parabolicity.	303
7A. Review 303 — 7B. Positive Superharmonic Functions 304 — 7C. Strict Inclusion $O_G < O_{HP}$ 304 — 7D. HP -Symmetry 306 — 7E. Characterization by Maximum Principle 308 — 7F. The Identity $C_\beta = O_G$ 309 — 7G. The Space $\mathcal{N}(R)$ 309 — 7H. The Class L 310 — 7I. Relative Classes 310 — 7J. The Class O_{HP}^n 311	
8. Green's Kernel	312
8A. Green's Function 312 — 8B. Behavior on Royden's Compactification 313 — 8C. Kernels 313 — 8D. Joint Continuity 314 — 8E. Local Behavior 314 — 8F. Extension to Royden's Compactification 314 — 8G. Green's Kernel on R_M^* 316 — 8H. Properties of $G(\cdot, \cdot)$ 316	
9. Green's Potentials	317
9A. Transfinite Diameter ρ 317 — 9B. Tchebycheff's Constant τ 318 — 9C. An Estimate 318 — 9D. Minimum Energy ε 320 — 9E. Maximum Principle 320 — 9F. Measure ξ 322 — 9G. Gauss' Variation 322 — 9H. Energy Principle 324 — 9I. Proof of Theorem 9D 325 — 9J. Uniqueness 325 — 9K. Dirichlet's Constant δ 326 — 9L. Identities 326	

10. Parabolicity Tests 328
 10A. Modular Test 328 — **10B.** Divergent Modular Product 328 —
 10C. Conformal Metric Test 329 — ***10D.** Euclidean Metric Test 330
 — **10E.** Regular Chain Test 331

11. Plane Regions. 332
 11A. Inclusion Relations 332 — **11B.** Removable Sets 333 — **11C.**
 Robin's Constant 334 — **11D.** Change under Conformal Mapping
 334 — **11E.** Logarithmic Potentials 335 — **11F.** Capacity of the Cantor
 Set 336

§ 3. Existence of Kernels 339

12. The Hyperbolic Case 340
 12A. The Fundamental Theorem 340 — **12B.** An Auxiliary Function
 340 — **12C.** An Inequality 342 — **12D.** Dirichlet Constant of $F_{n+1,m}$
 344 — **12E.** Proof of $\tau(\mathcal{E}_n) = \rho(\mathcal{E}_n) = \infty$ 344 — **12F.** Green's Potential
 on R_M^* 345 — **12G.** Evans' Potential 346 — **12H.** Reduction 347 —
 12I. An Estimate 347 — **12J.** Irregular Hyperbolic Surfaces 349 —
 12K. Green's Star Region 350

13. The Parabolic Case 351
 13A. Evans-Selberg Potential 351 — **13B.** Capacity Functions with
 Compact Level Lines 352 — **13C.** Existence Proof 352 — **13D.**
 Positive Singularities 353 — **13E.** Evans' Kernel 354 — **13F.** Proof
 355 — **13G.** Joint Continuity of Evans' Kernel 357 — **13H.** Joint Uni-
 form Convergence 359 — **13I.** The s -Kernel 361 — **13J.** Proof 362

Chapter VI • Functions with Iversen's Property

§ 1. Classes O_{A^0D} and O_{A^0B} 364

1. Iversen's Property 365
 1A. Functions with Iversen's Property 365 — **1B.** Cluster Set at the
 Ideal Boundary 366 — **1C.** Valence Function 367 — **1D.** Degree of
 a Component 368 — **1E.** Asymptotic Values 368 — **1F.** Deficiency
 on ∂W_{ij} 368 — **1G.** Totally Disconnected $C_R(f, \beta_S)$ 369 — **1H.** Total
 $C_R(f, \beta_S)$ 369 — **1I.** Stoilow's Principle 370 — **1J.** Proof of Theorem
 1B 370 — **1K.** Continuity of AB -Functions 370 — **1L.** Remark on
 Removable Sets 371

2. Meromorphic Functions on O_{A^0D} -Surfaces 372
 2A. Class MD^* 372 — **2B.** MD^* on O_{A^0D} -Surfaces 372 — **2C.**
 AD on O_{A^0D} -Surfaces 373 — **2D.** The Inclusion $O_{HD} < O_{A^0D}$ 374

3. Meromorphic Functions on O_{A^0B} -Surfaces 374
 3A. The Class M on O_{A^0B} -Surfaces 374 — **3B.** A Characterization
 of O_{A^0B} 375 — **3C.** Exceptional Sets 375 — **3D.** $AB \subset MD^*$ for O_{A^0B} -
 Surfaces 376 — **3E.** Integrated Form of O_{A^0B} -Test 377 — **3F.** Modular
 O_{A^0B} -Test 379 — **3G.** The Inclusion $O_{HB} < O_{A^0B}$ 380 — **3H.** Inclusion
 Relations 381

§ 2. Boundary Points of Positive Measure.	382
4. Abstraction.	383
4A. Relative X -Compactifications 383 — 4B. Points of Positive Measure 383 — 4C. Localizable U_X 384 — 4D. The Class $C_{X,Y}$ 384 — 4E. General Identity Theorem 385	
5. Identity Theorems on $O_{A^0 D^-}$ and $O_{A^0 B}$-Surfaces.	385
5A. The Class U_S 385 — 5B. The Class $U_S \cap O_{A^0 D}$ 386 — 5C. A Surface in $U_S \cap O_{A^0 D} - U_{\widehat{H}D}$ 386 — 5D. The Class $U_S \cap O_{A^0 B}$ 387 — 5E. A Surface in $U_S \cap O_{A^0 B} - U_{HB}$ 387	
6. The Class O_{MD^*}.	388
6A. Continuity on R_{MI}^* 388 — 6B. MD^* on U_{HD} -Surfaces 388 — 6C. M on U_{HB} -Surfaces 388 — 6D. A Criterion for O_{MD^*} 389 — 6E. Table of Strict Inclusion Relations 390	

Appendix. Higher Dimensions

1. Fundamentals.	392
1A. Riemannian Manifolds 392 — 1B. Differential Forms 393 — 1C. Harmonic Functions 395 — 1D. Compactifications 396	
2. Moduli of Annuli.	397
2A. Modulus 397 — 2B. Bisection into Small Annuli 397 — 2C. An Estimate 398 — 2D. Proof of the Theorem 399	
3. Parabolicity.	399
3A. Equivalences and Inclusions 399 — 3B. Parabolic Riemannian Ball 400 — 3C. Hyperbolic Punctured Torus 401 — 3D. The Equation $\Delta u = Pu$ ($P \geq 0$) 402 — 3E. Linear Operators 403 — 3F. Reduction to Fredholm's Equation 403 — 3G. Proof of $\ A\ _\infty < 1$ 404 — 3H. Proof of Hyperbolicity 405	
4. Invariance under Deformation.	405
4A. Inclusion Relations 405 — 4B. Wiener's and Royden's Mappings 406 — 4C. Quasiconformal Mappings 407 — 4D. Conformal Non-invariance 408 — 4E. Quasi-isometric Invariance 409 — 4F. Preliminaries 409 — 4G. Proof of Theorem 4E 410	
Bibliography.	412
Author Index.	437
Subject and Notation Index.	439