

Contents

PREFACE	v
CHAPTER I. SURFACE TOPOLOGY	
§1. TOPOLOGICAL SPACES	2
1. Definition	2
2. Subsets	3
3. Connectedness	5
4. Compactness	6
5. Countability	9
6. Mappings	11
7. Arcs	13
§2. SURFACES	14
8. Definitions	14
9. The fundamental group	15
10. The index of a curve	18
11. The degree of a mapping	20
12. Orientability	22
13. Bordered surfaces	23
§3. COVERING SURFACES	27
14. Smooth covering surfaces	27
15. The monodromy theorem	30
16. Applications of the monodromy theorem	32
17. The class of regular covering surfaces	34
18. The partial ordering	36
19. Cover transformations	37
20. Ramified covering surfaces	39
21. Complete covering surfaces	41
§4. SIMPLICIAL HOMOLOGY	44
22. Triangulations	44
23. Homology	48
24. Abelian groups	50
25. Polyhedrons	52
26. The 1-dimensional homology group	54
27. Relative homologies	55
28. Subdivisions	58
29. Exhaustions	61
30. Homologies on open polyhedrons	64
31. Intersection theory	67
32. Infinite cycles	72

§5. SINGULAR HOMOLOGY	75
33. Definitions and relation to the fundamental group	75
34. Simplicial approximation	78
35. Relative singular homology	80
§6. COMPACTIFICATION	81
36. Boundary components	81
37. The compactness proof	84
38. Partitions of the ideal boundary	87
§7. THE CLASSIFICATION OF POLYHEDRONS	90
39. Cell complexes	90
40. Canonical forms	94
41. Reduction to canonical forms	96
42. Topological classification	98
43. Determination of the fundamental group	99
44. Topological properties of open polyhedrons	102
§8. EXISTENCE OF TRIANGULATIONS	105
45. Coverings of finite character	105
46. The existence proof	107

CHAPTER II. RIEMANN SURFACES

§1. DEFINITIONS AND CONSTRUCTIONS	112
1. Conformal structure	112
2. Analytic mappings	115
3. Bordered Riemann surfaces	117
4. Riemann surfaces as covering surfaces	119
5. Metric structures	124
§2. ELEMENTARY THEORY OF FUNCTIONS ON RIEMANN SURFACES	127
6. Harmonic functions	127
7. The Dirichlet integral	130
8. Green's formula	132
9. Harnack's principle	134
10. Subharmonic functions	135
§3. THE DIRICHLET PROBLEM AND APPLICATIONS	138
11. The Dirichlet problem	138
12. Existence of a countable basis	142
13. Convergence in Dirichlet norm	146

CHAPTER III. HARMONIC FUNCTIONS ON RIEMANN SURFACES

§1. NORMAL OPERATORS	148
1. Linear classes of harmonic functions	149
2. Linear operators	150
3. The main existence theorem	154
4. Examples of normal operators	157

§2. PRINCIPAL OPERATORS	159
5. Operators on compact regions	160
6. Extremal harmonic functions on compact regions	161
7. Application to the principal operators	164
8. Extension to noncompact regions	166
§3. PRINCIPAL FUNCTIONS	169
9. Functions with singularities	169
10. Special cases	173
§4. THE CLASS OF PLANAR SURFACES	175
11. Slit mappings	176
12. The functions P_{hk}	181
13. Exponential mappings	185
§5. CAPACITIES	186
14. Capacity functions	186
15. The capacity of the boundary	188
16. Disk mappings	191

CHAPTER IV. CLASSIFICATION THEORY

§1. FUNDAMENTALS OF THE CLASSIFICATION THEORY	197
1. Basic properties of O_{HD} and O_{AD}	197
2. Planar surfaces of class O_{AD}	199
3. Connections between O_{AD} and O_{AB}	200
4. Removable singularities	201
5. Classes of harmonic functions	202
6. Parabolic and hyperbolic surfaces	203
7. Vanishing of the flux	206
§2. PARABOLIC AND HYPERBOLIC SUBREGIONS	209
8. Positive harmonic functions	209
9. Properties of the operator T	212
10. Properties connected with the Dirichlet integral	214
11. Functions with bounded means	216
§3. THE METHOD OF EXTREMAL LENGTH	219
12. Extremal length	220
13. Extremal distance	223
14. Conjugate extremal distance	227
§4. MODULAR TESTS	228
15. Harmonic modules	228
16. Analytic modules	230
§5. EXPLICIT TESTS	234
17. Deep coverings	234
18. Tests by triangulation	236

§6. APPLICATIONS	239
19. Regular covering surfaces	239
20. Ramified coverings of the sphere	243
§7. PLANE REGIONS	246
21. Mass distributions	246
22. The logarithmic potential	248
23. The classes O_{SB} and O_{SD}	250
§8. COUNTEREXAMPLES	251
24. The case of plane regions	252
25. Proof of $O_G < O_{HP} < O_{HB}$	256
26. Proof of $O_{HB} < O_{HD}$	261
CHAPTER V. DIFFERENTIALS ON RIEMANN SURFACES	
§1. ELEMENTARY PROPERTIES OF DIFFERENTIALS	265
1. Differential calculus	265
2. Integration	268
3. Conjugate differentials	270
4. The inner product	272
5. Differentials on bordered surfaces	274
6. Differentials on open surfaces	276
§2. THE METHOD OF ORTHOGONAL PROJECTION	277
7. The completion of Γ^1	277
8. The subclasses of Γ	280
9. Weyl's lemma	281
10. Orthogonal decompositions	282
11. Periods	284
12. Reproducing differentials	287
13. Compact bordered surfaces	288
14. Schottky differentials	291
15. Harmonic measures	294
16. Analytic differentials	296
§3. PERIODS AND SINGULARITIES	298
17. Singular differentials	298
18. Differentials of the second kind	300
19. Differentials and chains	305
20. Differentials and periods	310
21. The main existence theorem	311
22. Abel's theorem	315
23. The bilinear relation	317

§4. THE CLASSICAL THEORY	318
24. Analytic differentials on closed surfaces	319
25. Rational functions	321
26. Divisors	324
27. Riemann-Roch's theorem	325
28. Consequences of Riemann-Roch's theorem	329
 BIBLIOGRAPHY	 332
 INDEX	 374