

# Contents

<b>Introduction</b> .....	1
<b>Chapter I. Hopf Algebras</b> .....	8
1.1 Axioms of a Hopf Algebra .....	8
1.2 Group Algebras and Enveloping Algebras .....	15
1.3 Adjoint Action .....	20
1.4 The Hopf Dual .....	26
1.5 Comments and Complements .....	35
<b>Chapter 2. Excerpts from the Classical Theory</b> .....	36
2.1 Lie Algebras .....	36
2.2 Algebraic Lie Algebras .....	43
2.3 Algebraic Groups .....	48
2.4 Lie Algebras of Algebraic Groups .....	53
2.5 Comments and Complements .....	60
<b>Chapter 3. Encoding the Cartan Matrix</b> .....	62
3.1 Quantum Weyl Algebras .....	62
3.2 The Drinfeld Double .....	71
3.3 The Rosso Form and the Casimir Invariant .....	77
3.4 The Classical Limit and the Shapovalev Form .....	84
3.5 Comments and Complements .....	94
<b>Chapter 4. Highest Weight Modules</b> .....	96
4.1 The Jantzen Filtration and Sum Formula .....	96
4.2 Kac-Moody Lie Algebras .....	106
4.3 Integrable Modules for $U_q(\mathfrak{g}_C)$ .....	115
4.4 Demazure Modules and Product Formulae .....	121
4.5 Comments and Complements .....	128
<b>Chapter 5. The Crystal Basis</b> .....	131
5.1 Operators in the Crystal Limit .....	131
5.2 Crystals .....	136

5.3	Ad-invariant Filtrations, Twisted Actions and the Crystal Basis for $U_q(\mathfrak{n}^-)$ .....	141
5.4	The Grand Loop .....	150
5.5	Comments and Complements .....	160
<b>Chapter 6. The Global Bases</b> .....		161
6.1	The $\star$ Operation and the Embedding Theorem .....	161
6.2	Globalization .....	168
6.3	The Demazure Property .....	177
6.4	Littelmann's Path Crystals .....	185
6.5	Comments and Complements .....	198
<b>Chapter 7. Structure Theorems for <math>U_q(\mathfrak{g})</math></b> .....		201
7.1	Local Finiteness for the Adjoint Action .....	201
7.2	Positivity of the Rosso Form .....	215
7.3	The Separation Theorem .....	218
7.4	Noetherianity .....	223
7.5	Comments and Complements .....	229
<b>Chapter 8. The Primitive Spectrum of <math>U_q(\mathfrak{g})</math></b> .....		233
8.1	The Poincaré Series of the Harmonic Space .....	233
8.2	Factorization of the Quantum PRV Determinants .....	239
8.3	Verma Module Annihilators .....	245
8.4	Equivalence of Categories .....	251
8.5	Comments and Complements .....	259
<b>Chapter 9. Structure Theorems for <math>R_q[G]</math></b> .....		262
9.1	Commutativity Relations .....	262
9.2	Surjectivity and Injectivity Theorems .....	269
9.3	The Adjoint Action .....	276
9.4	The $R$ -Matrix .....	286
9.5	Comments and Complements .....	292
<b>Chapter 10. The Prime Spectrum of <math>R_q[G]</math></b> .....		294
10.1	Highest Weight Modules .....	294
10.2	The Quantum Weyl Group .....	304
10.3	Prime and Primitive Ideals of $R_q[G]$ .....	310
10.4	Hopf Algebra Automorphisms .....	320
10.5	Comments and Complements .....	324
<b>Appendix</b> .....		326
A.1	The Root System and the Weyl Group of a Cartan Matrix $C$ .....	326
A.2	Excerpts from Ring Theory .....	338

A.3 Combinatorial Identities and Dimension Theory ..... 345

A.4 Remarks on Constructions of Quantum Groups ..... 352

A.5 Comments and Complements ..... 361

**Bibliography** ..... 364

**Index of Notation** ..... 377

**Index** ..... 382