CONTENTS

PREFACE ix

PROLOGUE 1
- Tapping away in an evening at Djursholm 1
- What does an epsilon weigh? 1
- Red wine at the Stock Exchange Club 2
- Ice-cream in Madison 2
- Exact equality 3
- The deficiency of values 5
- Zurich 5
- Beautiful to look at, but... 6
- The unbearable ease of using norms 6
- Centenary Colloquium in Joensuu 7
- Two basic tasks, stability first 7
- And then accelerating the iteration 9
- Factoring the resolvent 10
- In the Hermann Weyl lecture hall 11
- A quiet life in Warsaw 12
- Finally, in Kirkkonummi 12

FIRST CHAPTER 15
- Resolvent 15
- Cauchy-integral 20

SECOND CHAPTER 23
- Entire functions 23
- Taylor coefficients 24
- Meromorphic functions 25
- The first main theorem 30
- Cartan's identity 31
- Order and type for meromorphic functions 32
- Boutroux-Cartan lemma 33
- Bound along a circle 34
- Representation theorems 36

THIRD CHAPTER 37
- Analytic vector valued functions 37
- Subharmonic functions 37
- Meromorphic vector valued functions 38

Rational functions 40
When is the inverse also meromorphic 42
A simple estimate for matrices 44

FOURTH CHAPTER **47**
A product form for matrices 47
Singular value decomposition 50
Basic inequalities for singular values and eigenvalues 51
The total logarithmic size of a matrix 54
Some basic properties of the total logarithmic size 56
Direct sum, Kronecker product and Hadamard product 60

FIFTH CHAPTER **63**
The total logarithmic size is subharmonic 63
Behavior near poles 65
Introducing T_1 for matrix valued functions 68
Basic identity for inversion 69
Extension to trace class 70
How to work outside the trace class 71

SIXTH CHAPTER **73**
Perturbation results 73
Special results for resolvents 77
Powers and their resolvents 79
Bounded characteristics 83
What if small perturbation means small in norm 85

SEVENTH CHAPTER **87**
Combining a scalar function with an operator 87
Representing F as G/g 93
Representations for the resolvent 94
Decay of spectral polynomials 96
Robust bounds for Krylov solvers 98
A bound for spectral projectors 100

EIGHTH CHAPTER **103**
Approximate polynomial degree of an analytic function 103
Some properties of the approximate polynomial degree 106
Approximate rational degree of a meromorphic function 108
Spijker's lemma 112
Power bounded operators and bounds for the Laurent coefficients 114

NINTH CHAPTER **117**
Growth of associated scalar functions 117
Locally algebraic and locally almost algebraic operators 121

TENTH CHAPTER **125**
Exceptional values 125
Simple asymptotics for resolvents of matrices 126
Eigenvalues and exceptional values 128
Deficient operators 131

EPILOGUE **133**
Lecturing and typing in Toronto 133
Fishing and finishing in Karjalohja 133

BIBLIOGRAPHY **135**