Contents

Foreword to the Third Edition ix

CHAPTER I. Holomorphic Functions of Several Variables 1
§1. Complex Euclidean space 1
1. The space $\mathbb{C}^n$ 1
2. The simplest domains 6
§2. Holomorphic functions 12
3. The concept of holomorphy 12
4. Pluriharmonic functions 15
5. Simplest properties of holomorphic functions 18
6. The fundamental theorem of Hartogs 24
§3. Series expansions 29
7. Power series 29
8. Other series 33
§4. Holomorphic mappings 39
9. Properties of holomorphic mappings 39
10. Biholomorphic mappings 44
11. Fatou's example 54

Problems 58

CHAPTER II. Basic Geometric Concepts 61
§5. Manifolds and Stokes's formula 61
12. The concept of a manifold 61
13. Complexification of Minkowski space 67
14. Stokes's formula 77
15. The Cauchy-Poincaré theorem 83
16. Maxwell's equations 86
§6. Geometry of the space $\mathbb{C}^n$ 96
17. Submanifolds of $\mathbb{C}^n$ 96
18. Wirtinger's theorem 101
19. The Fubini-Study form and related topics 108

§7. Coverings 112
20. The concept of a covering 112
21. Fundamental groups and coverings 115
22. Riemann domains 121
§8. Analytic sets 123
23. The Weierstrass preparation theorem 123
24. Properties of analytic sets 129
25. Local structure 136
§9. Bundles and sheaves 140
26. The concept of a bundle 140
27. The tangent and cotangent bundles 143
28. The concept of a sheaf 148

Problems 152

CHAPTER III. Analytic Continuation 155
§10. Integral representations 155
29. The formulas of Martinelli-Bochner and Leray 155
30. Weil's formula 161
§11. Extension theorems 166
31. Extension from the boundary 166
32. Hartogs's theorem and the removal of singularities 173
§12. Domains of holomorphy 177
33. The concept of a domain of holomorphy 177
34. Holomorphic convexity 181
35. Properties of domains of holomorphy 185
§13. Pseudoconvexity 188
36. The continuity principle 188
37. Local pseudoconvexity 192
38. Plurisubharmonic functions 199
39. Pseudoconvex domains 206
§14. Envelopes of holomorphy 212
40. One-sheeted envelopes 212
41. Multiple-sheeted envelopes 217
42. Analyticity of the set of singularities 223

Problems 228

CHAPTER IV. Meromorphic Functions and Residues 231
§15. Meromorphic functions 231
43. The concept of a meromorphic function 231
44. The first Cousin problem 234
45. Solution of the first problem 238

§16. Methods of sheaf theory 242
46. Cohomology groups 242
47. Exact sequences of sheaves 247
48. Localized first Cousin problem 249
49. Second Cousin problem 253
§17. Applications 259
50. Applications of the Cousin problems 259
51. Solution of the Levi problem 262
52. Other applications 264
§18. Higher-dimensional residues 272
53. The theory of Martinelli 272
54. The theory of Leray 278
55. Logarithmic residue 286

Problems 292

CHAPTER V. Some Problems of Geometric Function Theory 295
§19. Invariant metrics 295
56. The Bergman metric 295
57. The Carathéodory metric 302
58. The Kobayashi metric 305
§20. Hyperbolic manifolds 308
59. Criteria for hyperbolicity 308
60. Generalizations of Picard's theorem 317
§21. Boundary properties 327
61. Mappings of strictly pseudoconvex domains 327
62. Correspondence of boundaries 332
63. A symmetry principle 336
64. Vector fields 341
65. Boundary properties of functions 347
66. Uniqueness theorems and propositions 352

Problems 359

APPENDIX. Complex Potential Theory 361

Subject Index 369