

Contents

Introduction	1
1 Lie theory	3
1.1 Complex spaces	3
1.2 Lie group actions	6
1.3 One-parameter transformation groups	10
1.4 Vector fields	14
1.5 Infinitesimal transformations	16
1.6 Analyticity of Lie group actions	20
1.7 Lie homomorphism	22
1.8 Global actions	27
2 Automorphism groups	31
2.1 Topology in $\text{Hol}(X, Y)$	31
2.2 Local linearization of a compact group with a fixed point	35
2.3 The automorphism group of a compact complex space	39
2.4 Automorphisms of fiber bundles	44
2.5 Proper actions	48
2.6 The automorphism group of a bounded domain	50
2.7 The automorphism groups of the polydisk and the ball	54
2.8 A characterization of the ball	57
2.9 Bounded domains with compact quotient $D/\text{Aut}(D)$	61
3 Compact homogeneous manifolds	63
3.1 Flag manifolds	63
3.2 Equivariant projective embeddings	70
3.3 Automorphism groups of flag manifolds	73
3.4 Parallelizable manifolds	77
3.5 Tits fibration	79
3.6 Manifolds fibered by tori	82
3.7 The role of the fundamental group	85
3.8 An estimate of the dimension of $\text{Aut}(X)$	96
3.9 Compact homogeneous Kähler manifolds	100

4 Homogeneous vector bundles	105
4.1 Coherent analytic G -sheaves	105
4.2 Holomorphic vector G -bundles	110
4.3 Theorem of R.Bott. Proof of the Borel-Weil theorem	113
4.4 Application of the Leray spectral sequence	116
4.5 Proof of the theorem of R.Bott	118
4.6 Invertible sheaves on G/P for P maximal parabolic	121
4.7 Computations in root systems	125
4.8 Cohomology of the tangent sheaf	130
5 Function theory on homogeneous manifolds	135
5.1 Representations of compact Lie groups on Fréchet spaces	135
5.2 Differentiable vectors and Fourier series in $\mathcal{O}(X)$	144
5.3 Reductive complex Lie groups	147
5.4 Quasi-affine homogeneous varieties	153
5.5 Holomorphically separable homogeneous manifolds	157
5.6 Stein homogeneous manifolds	159
5.7 Observable subgroups	164
5.8 Invariant plurisubharmonic functions and geodesic convexity	171
Concluding remarks	181
Bibliography	186
Index of notations	196
Index of terminology	199