
Contents

Preface	xiii
Chapter 1. Selected Problems in One Complex Variable	1
§1.1. Preliminaries	2
§1.2. A Simple Problem	2
§1.3. Partitions of Unity	4
§1.4. The Cauchy-Riemann Equations	7
§1.5. The Proof of Proposition 1.2.2	10
§1.6. The Mittag-Leffler and Weierstrass Theorems	12
§1.7. Conclusions and Comments	16
Exercises	18
Chapter 2. Holomorphic Functions of Several Variables	23
§2.1. Cauchy's Formula and Power Series Expansions	23
§2.2. Hartog's Theorem	26
§2.3. The Cauchy-Riemann Equations	29
§2.4. Convergence Theorems	29
§2.5. Domains of Holomorphy	31
Exercises	35
Chapter 3. Local Rings and Varieties	37
§3.1. Rings of Germs of Holomorphic Functions	38
§3.2. Hilbert's Basis Theorem	39

§3.3. The Weierstrass Theorems	40
§3.4. The Local Ring of Holomorphic Functions is Noetherian	44
§3.5. Varieties	45
§3.6. Irreducible Varieties	49
§3.7. Implicit and Inverse Mapping Theorems	50
§3.8. Holomorphic Functions on a Subvariety	55
Exercises	57
Chapter 4. The Nullstellensatz	61
§4.1. Reduction to the Case of Prime Ideals	61
§4.2. Survey of Results on Ring and Field Extensions	62
§4.3. Hilbert's Nullstellensatz	68
§4.4. Finite Branched Holomorphic Covers	72
§4.5. The Nullstellensatz	79
§4.6. Morphisms of Germs of Varieties	87
Exercises	92
Chapter 5. Dimension	95
§5.1. Topological Dimension	95
§5.2. Subvarieties of Codimension 1	97
§5.3. Krull Dimension	99
§5.4. Tangential Dimension	100
§5.5. Dimension and Regularity	103
§5.6. Dimension of Algebraic Varieties	104
§5.7. Algebraic vs. Holomorphic Dimension	108
Exercises	110
Chapter 6. Homological Algebra	113
§6.1. Abelian Categories	113
§6.2. Complexes	119
§6.3. Injective and Projective Resolutions	122
§6.4. Higher Derived Functors	126
§6.5. Ext	131
§6.6. The Category of Modules, Tor	133
§6.7. Hilbert's Syzygy Theorem	137
Exercises	142

Chapter 7. Sheaves and Sheaf Cohomology	145
§7.1. Sheaves	145
§7.2. Morphisms of Sheaves	150
§7.3. Operations on Sheaves	152
§7.4. Sheaf Cohomology	157
§7.5. Classes of Acyclic Sheaves	163
§7.6. Ringed Spaces	168
§7.7. De Rham Cohomology	172
§7.8. Čech Cohomology	174
§7.9. Line Bundles and Čech Cohomology	180
Exercises	182
Chapter 8. Coherent Algebraic Sheaves	185
§8.1. Abstract Varieties	186
§8.2. Localization	189
§8.3. Coherent and Quasi-coherent Algebraic Sheaves	194
§8.4. Theorems of Artin-Rees and Krull	197
§8.5. The Vanishing Theorem for Quasi-coherent Sheaves	199
§8.6. Cohomological Characterization of Affine Varieties	200
§8.7. Morphisms – Direct and Inverse Image	204
§8.8. An Open Mapping Theorem	207
Exercises	212
Chapter 9. Coherent Analytic Sheaves	215
§9.1. Coherence in the Analytic Case	215
§9.2. Oka's Theorem	217
§9.3. Ideal Sheaves	221
§9.4. Coherent Sheaves on Varieties	225
§9.5. Morphisms between Coherent Sheaves	226
§9.6. Direct and Inverse Image	229
Exercises	234
Chapter 10. Stein Spaces	237
§10.1. Dolbeault Cohomology	237
§10.2. Chains of Syzygies	243
§10.3. Functional Analysis Preliminaries	245

§10.4. Cartan's Factorization Lemma	248
§10.5. Amalgamation of Syzygies	252
§10.6. Stein Spaces	257
Exercises	260
Chapter 11. Fréchet Sheaves – Cartan's Theorems	263
§11.1. Topological Vector Spaces	264
§11.2. The Topology of $\mathcal{H}(X)$	266
§11.3. Fréchet Sheaves	274
§11.4. Cartan's Theorems	277
§11.5. Applications of Cartan's Theorems	281
§11.6. Invertible Groups and Line Bundles	283
§11.7. Meromorphic Functions	284
§11.8. Holomorphic Functional Calculus	288
§11.9. Localization	298
§11.10. Coherent Sheaves on Compact Varieties	300
§11.11. Schwartz's Theorem	302
Exercises	309
Chapter 12. Projective Varieties	313
§12.1. Complex Projective Space	313
§12.2. Projective Space as an Algebraic and a Holomorphic Variety	314
§12.3. The Sheaves $\mathcal{O}(k)$ and $\mathcal{H}(k)$	317
§12.4. Applications of the Sheaves $\mathcal{O}(k)$	323
§12.5. Embeddings in Projective Space	325
Exercises	328
Chapter 13. Algebraic vs. Analytic – Serre's Theorems	331
§13.1. Faithfully Flat Ring Extensions	331
§13.2. Completion of Local Rings	334
§13.3. Local Rings of Algebraic vs. Holomorphic Functions	338
§13.4. The Algebraic to Holomorphic Functor	341
§13.5. Serre's Theorems	344
§13.6. Applications	351
Exercises	355

Chapter 14. Lie Groups and Their Representations	357
§14.1. Topological Groups	358
§14.2. Compact Topological Groups	363
§14.3. Lie Groups and Lie Algebras	376
§14.4. Lie Algebras	385
§14.5. Structure of Semisimple Lie Algebras	392
§14.6. Representations of $\mathfrak{sl}_2(\mathbb{C})$	400
§14.7. Representations of Semisimple Lie Algebras	404
§14.8. Compact Semisimple Groups	409
Exercises	416
Chapter 15. Algebraic Groups	419
§15.1. Algebraic Groups and Their Representations	419
§15.2. Quotients and Group Actions	423
§15.3. Existence of the Quotient	427
§15.4. Jordan Decomposition	430
§15.5. Tori	433
§15.6. Solvable Algebraic Groups	437
§15.7. Semisimple Groups and Borel Subgroups	442
§15.8. Complex Semisimple Lie Groups	451
Exercises	456
Chapter 16. The Borel-Weil-Bott Theorem	459
§16.1. Vector Bundles and Induced Representations	460
§16.2. Equivariant Line Bundles on the Flag Variety	464
§16.3. The Casimir Operator	469
§16.4. The Borel-Weil Theorem	474
§16.5. The Borel-Weil-Bott Theorem	478
§16.6. Consequences for Real Semisimple Lie Groups	483
§16.7. Infinite Dimensional Representations	484
Exercises	493
Bibliography	497
Index	501