

TABLE OF CONTENTS

ACKNOWLEDGEMENTS	vii
INTRODUCTION	1
<u>CHAPTER I. FIRST STEPS (1965-1970)</u>	5
<u>§1. An analysis of the method of Jacquet-Langlands</u>	5
1.1. Cuspidal representations and L -functions for $GL(2)$...	5
1.2. Global zeta-integrals and their factorization	8
1.3. The local zeta-integrals	11
1.4. More local theory	12
1.5. Global results for $L_S(s, \pi)$	15
1.6. Global results for $L(s, \pi)$	16
1.7. Description of the L -function machine	19
<u>§2. Eisenstein series and Langlands' Euler Products</u>	21
2.1. The example of $L(s, \chi)$	21
2.2. L -groups	23
2.3. Unramified representations	25
2.4. The general set-up: Preliminary definitions	30
2.5. The general set-up: Eisenstein series, constant terms, and Langlands' "Euler products"	33
<u>CHAPTER II. DEVELOPMENTS AND REFINEMENTS (1970-1982)</u>	41
<u>§1. Zeta-Integrals for $GL(n)$ and related groups</u>	41
1.1. The method of Tate-Godement-Jacquet	41
1.2. Jacquet's theory for $GL(2) \times GL(2)$ and the method of Rankin-Selberg	44
Appendix to Section (1.2): Analysis and reformulation of the method of Rankin-Selberg-Jacquet for $GL(2) \times GL(2)$	47
1.3. Shimura's method	59
1.4. Hecke theory for $GL(n)$	61
1.5. The metaplectic group	63

1.6.	Symmetric powers of L -functions	65
1.7.	$GL(n) \times GL(m)$	67
1.8.	Additional notes and references: L -functions and the lifting problem	70
1.9.	Concluding remarks	72
<u>§2.</u>	<u>Eisenstein series and generic representations</u>	75
2.1.	Whittaker models: General notions	75
2.2.	Whittaker models for $I(s, \pi_v)$	77
2.3.	Fourier coefficients of Eisenstein series	78
2.4.	Local coefficients and the functional equation for $L_S(s, \pi)$	79
2.5.	Examples	82
2.6.	On the non-vanishing of L -functions for $Re(s) = 1$	84
<u>CHAPTER III. RECENT DEVELOPMENTS (1982-)</u>		87
<u>§1.</u>	<u>Explicit Construction of Zeta-integrals á la Piatetski-Shapiro</u>	87
1.1.	Origins of the method of Piatetski-Shapiro and Rallis	87
1.2.	The construction of Piatetski-Shapiro and Rallis	90
1.3.	Summing up of the method	93
1.4.	Rankin triple products	94
1.5.	L -functions for $G \times GL(n)$	97
<u>§2.</u>	<u>Langlands Theory Completed</u>	102
2.1.	Range of applicability of the method	102
2.2.	A uniform line of convergence for $L_S(s, \pi, r)$	105
2.3.	Ramanujan-type estimates	107
2.4.	Analytic continuation of the completed L -function ..	109
2.5.	More examples	112
2.6.	On the uniqueness of local factors	113
	Last Words	116
<u>REFERENCES</u>		119
<u>INDEX</u>		129