

Contents

| | | |
|------------|---|-----------|
| I | p-adic Modular Forms | 1 |
| I.1 | Level Structures and Trivializations | 3 |
| I.2 | p -adic Modular Forms with Growth Conditions | 4 |
| I.2.1 | Definitions | 5 |
| I.2.2 | Basic Properties | 7 |
| I.3 | Generalized p -adic Modular Functions | 13 |
| I.3.1 | Definition | 14 |
| I.3.2 | The q -expansion map | 16 |
| I.3.3 | Diamond operators | 17 |
| I.3.4 | Weight and nebentypus | 19 |
| I.3.5 | Modular forms and modular functions | 19 |
| I.3.6 | Divided congruences | 23 |
| I.3.7 | Appendix: Modular forms of weight χ | 26 |
| II | Hecke and U Operators | 30 |
| II.1 | Hecke Operators | 30 |
| II.1.1 | Direct definition | 31 |
| II.1.2 | Hecke operators on divided congruences | 33 |
| II.2 | The Frobenius Operator | 34 |
| II.3 | The U Operator | 42 |
| II.3.1 | Definition | 42 |
| II.3.2 | U and overconvergence | 43 |
| II.3.3 | U and Frobenius | 51 |
| II.3.4 | Spectral theory: the overconvergent case | 54 |
| II.3.5 | Spectral theory: the ordinary case | 59 |
| II.3.6 | The characteristic power series | 61 |
| II.3.7 | Varying the weight | 66 |
| II.4 | Appendix: Hida's theory of the ordinary part | 69 |
| III | Galois Representations | 71 |
| III.1 | Duality Theorems | 72 |
| III.1.1 | Classical duality | 72 |
| III.1.2 | Duality for parabolic p -adic modular functions | 73 |

| | |
|--|-----|
| III.1.3 The non-parabolic case | 77 |
| III.2 Families of Modular Forms | 80 |
| III.3 Changing the Level | 83 |
| III.4 Deformations of Residual Eigenforms | 87 |
| III.4.1 Universal deformations | 88 |
| III.4.2 Deformations outside Np | 89 |
| III.4.3 Some classical results | 92 |
| III.5 Deformations of Galois Representations | 93 |
| III.6 The modular deformation space | 99 |
| III.6.1 Changing the weight | 99 |
| III.6.2 Twisting | 100 |
| III.6.3 Families of twists, and an estimate for the Krull dimension of the modular deformation ring | 106 |
| III.6.4 The ordinary case | 109 |
| III.7 Further Questions | 112 |