

Contents

<i>Preface</i>	<i>page</i>	xi
<i>List of notation</i>		xiii
1 Dirichlet series: I	1	
1.1 Generating functions and asymptotics	1	
1.2 Analytic properties of Dirichlet series	11	
1.3 Euler products and the zeta function	19	
1.4 Notes	31	
1.5 References	33	
2 The elementary theory of arithmetic functions	35	
2.1 Mean values	35	
2.2 The prime number estimates of Chebyshev and of Mertens	46	
2.3 Applications to arithmetic functions	54	
2.4 The distribution of $\Omega(n) - \omega(n)$	65	
2.5 Notes	68	
2.6 References	71	
3 Principles and first examples of sieve methods	76	
3.1 Initiation	76	
3.2 The Selberg lambda-squared method	82	
3.3 Sifting an arithmetic progression	89	
3.4 Twin primes	91	
3.5 Notes	101	
3.6 References	104	
4 Primes in arithmetic progressions: I	108	
4.1 Additive characters	108	
4.2 Dirichlet characters	115	
4.3 Dirichlet L -functions	120	

4.4	Notes	133
4.5	References	134
5	Dirichlet series: II	137
5.1	The inverse Mellin transform	137
5.2	Summability	147
5.3	Notes	162
5.4	References	164
6	The Prime Number Theorem	168
6.1	A zero-free region	168
6.2	The Prime Number Theorem	179
6.3	Notes	192
6.4	References	195
7	Applications of the Prime Number Theorem	199
7.1	Numbers composed of small primes	199
7.2	Numbers composed of large primes	215
7.3	Primes in short intervals	220
7.4	Numbers composed of a prescribed number of primes	228
7.5	Notes	239
7.6	References	241
8	Further discussion of the Prime Number Theorem	244
8.1	Relations equivalent to the Prime Number Theorem	244
8.2	An elementary proof of the Prime Number Theorem	250
8.3	The Wiener–Ikehara Tauberian theorem	259
8.4	Beurling’s generalized prime numbers	266
8.5	Notes	276
8.6	References	279
9	Primitive characters and Gauss sums	282
9.1	Primitive characters	282
9.2	Gauss sums	286
9.3	Quadratic characters	295
9.4	Incomplete character sums	306
9.5	Notes	321
9.6	References	323
10	Analytic properties of the zeta function and L-functions	326
10.1	Functional equations and analytic continuation	326
10.2	Products and sums over zeros	345
10.3	Notes	356
10.4	References	356

11 Primes in arithmetic progressions: II	358
11.1 A zero-free region	358
11.2 Exceptional zeros	367
11.3 The Prime Number Theorem for arithmetic progressions	377
11.4 Applications	386
11.5 Notes	391
11.6 References	393
12 Explicit formulæ	397
12.1 Classical formulæ	397
12.2 Weil's explicit formula	410
12.3 Notes	416
12.4 References	417
13 Conditional estimates	419
13.1 Estimates for primes	419
13.2 Estimates for the zeta function	433
13.3 Notes	447
13.4 References	449
14 Zeros	452
14.1 General distribution of the zeros	452
14.2 Zeros on the critical line	456
14.3 Notes	460
14.4 References	461
15 Oscillations of error terms	463
15.1 Applications of Landau's theorem	463
15.2 The error term in the Prime Number Theorem	475
15.3 Notes	482
15.4 References	484
APPENDICES	
A The Riemann–Stieltjes integral	486
A.1 Notes	492
A.2 References	493
B Bernoulli numbers and the Euler–MacLaurin summation formula	495
B.1 Notes	513
B.2 References	517

C The gamma function	520
C.1 Notes	531
C.2 References	533
D Topics in harmonic analysis	535
D.1 Pointwise convergence of Fourier series	535
D.2 The Poisson summation formula	538
D.3 Notes	542
D.4 References	542
<i>Name index</i>	544
<i>Subject index</i>	550