
Contents

Preface	ix
1 An Introduction to the Basic Problem	1
1.1 The Problem Posed	1
1.2 Interpretation of Integral Equations of the First Kind . .	3
1.3 Some Ground Rules	3
1.4 A Minor Difficulty Overcome	4
1.5 Summary	4
Problems I	5
2 Some Examples	7
2.1 Introduction	7
2.2 The Laplace Transform	7
2.3 Some Remarks About Transport Theory	8
2.4 Determination of a Signal by Absorption—Slab Geometry	9
2.5 X-Ray Determination of the Density Distribution in a Sphere—PHERMEX Problem	11
2.6 The Problem of Tomography	13
2.7 Determination of Source Strength in an Absorbing Material	15
2.8 Determination of the Radial Density Distribution of Plutonium in a Small Sphere	16
2.9 A Problem in Geomagnetic Prospecting	17
2.10 A Problem for the Department of Game and Fish	18

2.11	A Problem in Mechanics	19
2.12	Some Observations and an Important Concept	21
2.13	Summary	22
	Problems II	22
3	A Bit of Functional Analysis	27
3.1	Introduction	27
3.2	Norms of Functions	27
3.3	Norms of Vectors	29
3.4	Norms of Integral Operators	30
3.5	Norms of Differential Operators	32
3.6	Norms of Matrix Operators	35
3.7	Norms of General Operators T	36
3.8	Summary	37
	Problems III	38
4	Integral Operators with Separable Kernels	41
4.1	Introduction	41
4.2	Separable Kernels	41
4.3	Some Properties of Integral Operators with Separable Kernels	42
4.4	IFKs with Separable Kernels	43
4.5	Eigenvalues and Eigenfunctions of Integral Operators with Separable Kernels	45
4.6	Integral Equations of the Second Kind with Separable Kernels	46
4.7	Summary	48
	Problems IV	48
5	Integral Operators with General Kernels	51
5.1	Introduction	51
5.2	Some More Facts About Integral Operators—Existence and Boundedness of Inverses	52
5.3	Eigenvalues and Eigenfunctions of Integral Operators with Symmetric Kernels	56
5.4	Eigenvalues and Eigenfunctions of Nonsymmetric Operators—Singular Values and Singular Functions	58
5.5	Integral Equations of the Second Kind with Symmetric Kernels	63

5.6	Integral Equations of the First Kind with General Kernels	65
5.7	Summary	66
	Problems V	67
6	Some Methods of Resolving Integral Equations of the First Kind	71
6.1	Introduction	71
6.2	Classical Quadrature Approach	72
6.2.1	Basic Scheme	72
6.2.2	A Particular Application of the Quadrature Approach	73
6.2.3	A Trivial Example and Some Important Observations	76
6.2.4	Section 6.2.2 Revisited	78
6.2.5	A Return to the Kernel $K = 1 + \eta xy$	78
6.2.6	The Condition Number of a Matrix	79
6.2.7	A Digression Concerning Approximate Kernels	80
6.2.8	The Method of Least Squares	81
6.2.9	Some Miscellaneous Comments Concerning the Quadrature Method	83
6.2.10	A Summary of the Classical Quadrature Approach	85
6.3	General Series Expansions	85
6.3.1	Basic Scheme	85
6.3.2	Series Expansions and Projection and Collocation Methods	87
6.4	Expansion in Series of Eigenfunctions and Singular Functions	89
6.4.1	Basic Scheme	89
6.4.2	A Variant of the Method	90
6.5	Parametrized Solution Function Method	91
6.6	The Method of Regularization	92
6.6.1	Some Background and the Basic Scheme	92
6.6.2	Some Numerical Examples	97
6.6.3	The Selection of the Parameter, γ	103
6.6.4	A Generalization of the Concept of Regularization	103
6.7	Stochastic Approaches	103
6.8	Iterative Methods	106
6.9	Summary	107
	Problems VI	107

7	Some Important Miscellany	113
7.1	Introduction	113
7.2	About Those Ground Rules	113
7.3	A Final Word About Codes and Quadrature Schemes	114
7.4	Finding Generalized Moments of Solutions	115
7.5	IFKs in More Variables	116
7.6	Errors in the Kernel	117
7.7	Summary	117
	Problems VII	117
8	Epilogue	119
	References	121
	Appendix A	125
	Appendix B	127
	Appendix C	129
	Index	133