

# Contents

<b>Preface</b> .....	XI
<b>Introduction</b> .....	XIII
<b>Guidelines for the Reader</b> .....	XV
<b>Chapter 1    Introductory Material</b>	
1.1    On the Evolution of the B.I.E.M./B.E.M. ....	1
1.2    Elements of Nonsmooth Analysis .....	5
1.2.1    Elements of Nonsmooth-Convex Analysis .....	5
1.2.2    Elements of Nonsmooth-Nonconvex Analysis .....	13
1.3    Contact Problems. ....	18
1.3.1    Monotone Multivalued Boundary and Interface Conditions. Pointwise Formulations .....	18
1.3.2    Extensions of the Monotone Multivalued Boundary Conditions to Function Spaces .....	25
1.3.3    Nonmonotone Multivalued Boundary Conditions .....	30
1.4    Bilateral and Unilateral Problems .....	34
1.4.1    Variational Formulations .....	34
1.5    Existence Results for Variational and Hemivariational Inequalities .....	37
<b>Chapter 2    The Direct and Indirect B.I.E.M. for Bilateral Problems</b>	
2.1    The B.V.P. of Linear Elasticity .....	43
2.2    The Method of Weighted Residuals .....	44
2.3    Generalized Variational Principles .....	48
2.4    The Use of Reciprocal Theorems .....	49
2.5    The Singularity Method (Indirect Method) .....	53
<b>Chapter 3    Boundary Integral Formulations for Some Special Elastostatic B.V.Ps</b>	
3.1    Bending of Beams and Stretching of Bars .....	65
3.2    A Direct B.I.E.M. for Kirchhoff Plates .....	71
3.3    A Direct B.I.E.M. for Reissner Plates .....	76
<b>Chapter 4    On the Numerical Implementation of Boundary Element Equations</b>	
4.1    General Methods .....	85
4.2    Kirchhoff Plate Boundary Element Equations by the Point Collocation Method .....	87
4.3    The Galerkin B.E.M. in Reissner Plate Theory .....	89

**Chapter 5 Extension to Dynamic Problems**

5.1	Generalities .....	91
5.2	Steady State and Harmonic Problems .....	93
5.3	Numerical Applications .....	96
5.3.1	Convergence Studies and Cut-off Errors .....	96
5.3.2	Noise Distribution Around 2-D Barrier Models .....	101
5.4	Time Domain Formulation for Transient Problems .....	105
5.4.1	Wave Propagation in 2-D Elastic Media .....	105
5.4.2	Sound Pressure Waves in 3-D Acoustics and Numerical Applications .....	110

**Chapter 6 Dynamic Interaction Problems**

6.1	Bilateral Coupling of Elastic Structures and Domains .....	119
6.2	Fluid-Structure Interaction .....	127
6.3	Unilateral Contact Problems .....	132
6.3.1	Dynamical Problems. The Trial and Error Method .....	132
6.3.2	Examples: Elastic Massive Foundations on Elastic .....	138

**Chapter 7 B.I. Formulations for the Signorini-Fichera Inequality Problem**

7.1	Primal, Dual and Mixed Formulations of the B.V.P. ....	147
7.2	Integral Formulation with Respect to the Traction of the Contact Area .....	150
7.3	Integral Formulation with Respect to the Displacements of the Contact Area .....	153
7.4	The numerical Treatment .....	159

**Chapter 8 Mathematical Study of the B.I. Formulations of the Signorini-Fichera B.V.P.**

8.1	The Signorini-Fichera B.V.P: The Multivalued B.I.E with Respect to the Boundary Displacements .....	165
8.2	The Signorini-Fichera B.V.P: The Multivalued B.I.E with Respect to the Boundary Traction .....	172

**Chapter 9 Boundary Integral Formulation of the Frictional Unilateral Contact B.V.P.**

9.1	The Signorini Problem with Given Friction. Primal Problem, Mixed Problem and Approximation Results .....	177
9.2	The Derivation of a Multivalued B.I.E. for the Signorini Problem with Given Friction .....	180
9.3	On the Coulomb's Friction Problem. Numerical Results .....	188

<b>Chapter 10 Boundary Integral Formulations for the Monotone Multivalued Boundary Conditions</b>	
10.1	Convex Problems. Primal, Dual and Mixed Problems ..... 193
10.2	The Multivalued B.I.E. with Respect to the Boundary Traction on $\Gamma_3$ ..... 196
10.3	The Multivalued B.I.E. with Respect to the Displacements $u$ on $\Gamma_3$ ..... 199
10.4	Certain Semicoercive Multivalued B.I.Es. Existence Results ..... 205
<b>Chapter 11 Elastodynamic Unilateral Problems. A B.I.E. Approach</b>	
11.1	The Time Discretization Scheme. Time-Difference Multivalued B.I.Es. .... 209
11.2	Numerical Applications ..... 213
<b>Chapter 12 Nonconvex Unilateral Contact Problems</b>	
12.1	A Boundary Integral Equation with Respect to the Boundary Traction ..... 223
12.2	A Multivalued Boundary Integral Formulation with Respect to the Displacements on $\Gamma_3$ . .... 227
12.3	On the Numerical Treatment of Nonmonotone (Zigzag) Multivalued Contact Laws. A New Efficient Algorithm. .... 230
12.4	A Numerical Application: The Nonmonotone Friction and the Adhesive Contact Problem with Debonding. A Fixed Point Type Algorithm. .... 237
12.5	A Short Note on Certain Coercive and Semicoercive Nonconvex Unilateral Contact Problems. .... 246
<b>Chapter 13 Miscellanea</b>	
13.1	Unilateral Contact and Friction in Cracks. A General Indirect B.I.E.M. for Inequality Problems ..... 247
13.2	Debonding and Delamination in Adhesively Bonded Cracks. .... 250
13.3	Fractal Interfaces and Boundaries. .... 251
13.4	A Neurocomputing Approach to the Multivalued B.I.Es of the Inequality Contact Problems ..... 258
13.5	A Supervised Learning Approach to the Parameter Identification in Contact Problems ..... 261
<b>References</b> ..... 263	
<b>Subject Index</b> ..... 305	