

Contents

| | |
|---|-----------|
| Preface | ix |
| Introduction | 1 |
| 1 General problems of regularizability | 4 |
| 1.1 Definition of regularizing algorithm (RA) | 4 |
| 1.2 General theorems on regularizability and principles of constructing the regularizing algorithms | 7 |
| 1.3 Estimates of approximation error in solving the ill-posed problems | 16 |
| 1.4 Comparison of RA. The concept of optimal algorithm | 19 |
| 2 Regularizing algorithms on compacta | 23 |
| 2.1 The normal solvability of operator equations | 24 |
| 2.2 Theorems on stability of the inverse mappings | 26 |
| 2.3 Quasisolutions of the ill-posed problems | 28 |
| 2.4 Properties of δ -quasisolutions on the sets with special structure | 36 |
| 2.5 Numerical algorithms for approximate solving the ill-posed problem on the sets with special structure | 40 |
| 3 Tikhonov's scheme for constructing regularizing algorithms | 43 |
| 3.1 RA in Tikhonov's scheme with a priori choice of the regularization parameter | 43 |
| 3.2 A choice of regularization parameter with the use of the generalized discrepancy | 47 |
| 3.3 Application of Tikhonov's scheme to Fredholm integral equations of the first kind | 57 |
| 3.4 Tikhonov's scheme for nonlinear operator equations | 61 |
| 3.5 Numerical implementation of Tikhonov's scheme for solving operator equation | 68 |

| | | |
|----------|---|------------|
| 4 | General technique for constructing linear RA for linear problems in Hilbert space | 73 |
| 4.1 | General scheme for constructing RA for linear problems with completely continuous operator | 74 |
| 4.2 | General case of constructing the approximating families and RA | 77 |
| 4.3 | Error estimates for solutions of the ill-posed problems. The optimal algorithms | 90 |
| 4.4 | Regularization in case of perturbed operator | 100 |
| 4.5 | Construction of linear approximating families and RA in Banach space | 114 |
| 4.6 | Stochastic errors. Approximation and regularization of the solution of linear problems in case of stochastic errors | 122 |
| 5 | Iterative algorithms for solving non-linear ill-posed problems with monotonic operators. Principle of iterative regularization | 127 |
| 5.1 | Variational inequalities as a way of formulating non-linear problems | 128 |
| 5.2 | Equivalent transforms of variational inequalities | 131 |
| 5.3 | Browder-Tikhonov approximation for the solutions of variational inequalities | 136 |
| 5.4 | Principle of iterative regularization | 141 |
| 5.5 | Iterative regularization based on the zero-order techniques . . | 142 |
| 5.6 | Iterative regularization based on the first-order technique (regularized Newton technique) | 150 |
| 5.7 | RA for solving variational inequalities | 155 |
| 5.8 | Estimates of convergence rate of the iterative regularizing algorithms | 160 |
| 6 | Applications of the principle of iterative regularization | 164 |
| 6.1 | Algorithms for minimizing convex functionals. Solving the non-linear equations with monotonic operators | 164 |
| 6.2 | Algorithms for minimizing quadratic functionals. Non-linear procedures for solving linear problems | 168 |
| 6.3 | Iterative algorithms for solving general problems of mathematical programming | 172 |
| 6.4 | Algorithms to find the saddle points and equilibrium points in games | 178 |

| | | |
|----------|---|------------|
| 7 | Iterative methods for solving non-linear ill-posed operator equations with non-monotonic operators | 185 |
| 7.1 | Iteratively regularized Gauss – Newton technique for operator equations | 186 |
| 7.2 | The other ways of constructing iterative algorithms for general ill-posed operator equations | 195 |
| 8 | Application of regularizing algorithms to solving practical problems | 199 |
| 8.1 | Inverse problems of image processing | 200 |
| 8.2 | Reconstructive computerized tomography | 208 |
| 8.3 | Computerized tomography of layered objects | 213 |
| 8.4 | Tomographic examination of objects with focused radiation . | 218 |
| 8.5 | Seismic tomography in engineering geophysics | 222 |
| 8.6 | Inverse problems of acoustic sounding in wave approximation | 227 |
| 8.7 | Inverse problems of gravimetry | 236 |
| 8.8 | Problems of linear programming | 238 |
| | Bibliography | 242 |
| | Index | 254 |