TABLE OF CONTENTS

INTRODUCTION IX

PART I

CHAPTER 1 - Historic Sketch 1

CHAPTER 2 - Bessel Polynomials and Bessel Functions 4
Differential equations, their d-forms and their δ-forms. Polynomial solutions. Their relations to Bessel functions. Generalized Bessel Polynomials.

CHAPTER 3 - Recurrence Relations 18
Recurrence relations for y_n, θ_n, ϕ_n. Representation of BP by determinants. Recurrence relations for the generalized polynomials.

CHAPTER 4 - Moments and Orthogonality on the Unit Circle 25
Moment problems and solutions by Stieltjes, Tchebycheff, Hamburger; the Bessel alternative. Weight function of the generalized BP. Moments of the simple BP. Orthogonality on the unit circle.

PART II

CHAPTER 5 - Relations of the BP to the classical orthonormal polynomials and to other functions 34
BP as generalized hypergeometric functions, as limits of Jacobi Polynomials, as Laguerre Polynomials; their representation by Whittaker functions and by Lommel Polynomials.

CHAPTER 6 - Generating Functions 41
Generating functions and pseudogenerating functions. Results of Krall and Frink, Burchnall, Al-Salam, Brafman, Carlitz, and others. The theory of Lie groups and generating functions. Results of Weisner, Chatterjea, Das, McBride, Chen and Feng, and others. Different types of generating functions.

CHAPTER 7 - Formulas of Rodrigues Type 51
Methods of differential operators, of moments and of generating functions. Combinatorial Lemmas.

CHAPTER 8 - The BP and Continued Fractions 59
The BP as partial quotients. Approximation of the exponential function by ratios of BP.

CHAPTER 9 - Expansions of functions in series of BP 64
Formal expansions in series of the polynomials $y_n(z;a,b)$, or $\theta_n(z;a,b)$. The Boas-Buck theory of generalized Appell Polynomials. Convergence and summability of expansions in BP. Applications to expansions of powers and of exponentials.

PART III

CHAPTER 10 - Properties of the zeros of BP 75
Location of zeros. Results of Burchnall, Grosswald, Dickinson, Agarwal, Barnes, van Rossum, Nasif, Parodi, McCarthy, Dočev, Wragg and Underhill, Saff and Varga. Olver's theorem. Laguerre's Theorem. Results of Ismail and Kelker. Sums of powers of the zeros.

CHAPTER 11 - On the algebraic irreducibility of the BP 99
Theorems of Dumas, Eisenstein, and Breusch. Newton Polygon. Degrees of possible factors. Cases of irreducibility. Schemes of factorization. Two conjectures.

CHAPTER 12 - The Galois Group of BP 116
Theorems of Schur, Dedekind, Jordan, Cauchy, and Burnside. Resolvent and Discriminant. The Galois Group of the irreducible BP is the symmetric group. Details of the case n = 8.

CHAPTER 13 - Asymptotic properties of the BP 124
Case of n constant, $z \to 0$. Case of constant z, $n \to \infty$. Results of Grosswald, Obreshkov, Dočev.

PART IV

CHAPTER 14 - Applications 131
The irrationality of e^r (r rational) and of π^2. Solution of the wave equation. The infinite divisibility of the Student t-distribution. Bernstein's theorem. Electrical networks with maximally flat delay. The inversion of the Laplace transform. Salzer's theorem.

CHAPTER 15 - Miscellanea 150
Mention of the work by many authors, not discussed in the preceding chapters.

APPENDIX - Some open problems related to BP 162

BIBLIOGRAPHY of books and papers related to BP 164

BIBLIOGRAPHY of literature not directly related to BP 171

SUBJECT INDEX 175

NAME INDEX 179

PARTIAL LIST OF SYMBOLS 181