

Contents

Chapter I

The Riemann-Roch Theorem

§1. Lemmas on Valuations	1
§2. The Riemann-Roch Theorem	5
§3. Remarks on Differential Forms	14
§4. Residues in Power Series Fields	16
§5. The Sum of the Residues	21
§6. The Genus Formula of Hurwitz	26
§7. Examples	27
§8. Differentials of Second Kind	29
§9. Function Fields and Curves	31
§10. Divisor Classes	34

Chapter II

The Fermat Curve

§1. The Genus	36
§2. Differentials	37
§3. Rational Images of the Fermat Curve	39
§4. Decomposition of the Divisor Classes	43

Chapter III

The Riemann Surface

§1. Topology and Analytic Structure	46
§2. Integration on the Riemann Surface	51

Chapter IV

The Theorem of Abel-Jacobi

§1. Abelian Integrals	54
§2. Abel's Theorem	58
§3. Jacobi's Theorem	63
§4. Riemann's Relations	66
§5. Duality	67

Chapter V

Periods on the Fermat Curve

§1. The Logarithm Symbol	73
§2. Periods on the Universal Covering Space	75
§3. Periods on the Fermat Curve	77
§4. Periods on the Related Curves	81

Chapter VI

Linear Theory of Theta Functions

§1. Associated Linear Forms	83
§2. Degenerate Theta Functions	89
§3. Dimension of the Space of Theta Functions	90
§4. Abelian Functions and Riemann-Roch Theorem on the Torus	97
§5. Translations of Theta Functions	101
§6. Projective Embedding	104

Chapter VII

Homomorphisms and Duality

§1. The Complex and Rational Representations	110
§2. Rational and p -adic Representations	113
§3. Homomorphisms	116
§4. Complete Reducibility of Poincaré	117
§5. The Dual Abelian Manifold	118
§6. Relations with Theta Functions	121
§7. The Kummer Pairing	124
§8. Periods and Homology	127

Chapter VIII

Riemann Matrices and Classical Theta Functions

§1. Riemann Matrices	131
§2. The Siegel Upper Half Space	135
§3. Fundamental Theta Functions	138

Chapter IX

Involutions and Abelian Manifolds of Quaternion Type

§1. Involutions 143
 §2. Special Generators 146
 §3. Orders 148
 §4. Lattices and Riemann Forms on \mathbf{C}^2 Determined
 by Quaternion Algebras 149
 §5. Isomorphism Classes 154

Chapter X

Theta Functions and Divisors

§1. Positive Divisors 157
 §2. Arbitrary Divisors 163
 §3. Existence of a Riemann Form on an Abelian Variety 163

Bibliography 165

Index 167