Chapter 1

The Gamma Function and the Beta Function

Introduction, Table and Graph

Integral Expressions of $\Gamma(x)$ Probs. 1-3

Properties of $\Gamma(x)$ Probs. 4, 10, 12-21, 31 Specific Evaluations of $\Gamma(x)$ Probs. 5-9, 11, 22, 23

Infinite Product Expression of $\Gamma(x)$ Prob. 24 $\Gamma'(1) = \text{Negative of Euler's Constant}$ Prob. 32

Logarithmic Derivative of $\Gamma(x)$ for Positive Integers *Prob.* 33

Integral Expressions of B(x, y) Probs. 25, 27, 28 Properties of B(x, y) Probs. 26, 29, 30

Table of Formulas

Chapter II

Applications of the Gamma Function and the Beta Function

Introduction

Evaluation of Integrals Probs. 1-11, 14-27, 33, 35

 $\Gamma(\frac{1}{2}) = \sqrt{\pi}$ Probs. 12, 13
Infinite Product Expression of $\pi/2$ Probs. 28, 29

Evaluation of Certain Geometrical Magnitudes Probs. 30-32, 34, 36-39 Evaluation of Certain Physical Quantities Probs. 36, 40-42

Approximation of n! for Large Integers

Two Problems in Probability

A Problem in Heat Flow in a Straight Wire

Prob. 50

Chapter III

Legendre Polynomials

Introduction

Coefficients in Expansion of a Generating Function Prob. 1

Recurrence Relations Probs. 2, 13–15, 23, 24 Laplace's Integral Expression of $P_n(x)$ Prob. 3

Determination of Specific Legendre Polynomials

Prob. 4-9

Rodrigues's Expression for $P_n(x)$ Prob. 10 Orthogonality Property and Related Property Probs. 16, 17

Expansion of a Given Function in Legendre Polynomials Probs. 11, 18, 19

 $|P_n(x)| \le 1 \text{ for } -1 \le x \le 1$ Prob. 12

Character and Location of Zeros of $P_n(x)$ Prob. 21

Evaluation of Integrals Involving Legendre

Polynomials *Probs. 20, 26, 28–36*

Evaluation of $P_{2n}(0)$ Prob. 25

vi Contents

Evaluation of Derivatives

Probs. 22, 25, 27

Tables and Graph

Chapter IV

Applications of Legendre Polynomials

Introduction

Specific Series Expansions Probs. 1–8, 16, 17

Steady-state Heat-flow Temperature Distribution Probs. 9-13

Gravitational Potential of a Circular Lamina Prob. 14
Potential of an Electric Charge Distribution Prob. 15
Infinite Product Expression for $\pi/2$ Prob. 18

Application of Gauss's Mechanical Quadrature

Formula with Pertinent Table Probs. 19–20

Chapter V

Bessel Functions

Introduction

Differentiation Formulas Probs. 1-6, 8-10, 12, 15, 19

Recursion Formulas

Specific Evaluations

Generating Functions

Orthogonality Property and Related Property

Probs. 22, 23

Probs. 24, 25

Orthogonality Property and Related Property Probs. 24, 25
Expansion of a Given Function in Bessel Functions Probs. 26, 27

Evaluation of Integrals Involving Bessel Functions

Probs. 20, 27

Probs. 11, 28–33

Functions of Orders $n + \frac{1}{2}$ Probs. 17, 18, 20
Alternation of Zeros Prob. 21

Alternation of Zeros Prob. 21
Approximations for Small and Large Arguments Probs. 34, 35

Integral Expression of $J_p(x)$ and of $J_n(x)$ Probs. 36, 37 Relations to Legendre Polynomials Probs. 38-41

Formulas and Tables

Chapter VI

Applications of Bessel Functions

Introduction

Solutions of Equations Reducible to Bessel's Equation Probs. 1-6

Specific Expansions in Bessel Functions
Problems in Dynamics
Problems in Dynamics
Problems in Dynamics

Problems in Dynamics Probs. 11-17, 21, 22
Flux Distribution in a Nuclear Reactor Prob 18

Heat-flow Temperature Distribution

Probs. 19 20 24 25 30

Fluid Velocity Imparted by Radially Pulsating Cylinder Prob. 23

Displacement of Vibrating Annular Membrane Prob. 26
Alternating Current Density Prob. 27
Eddy Current Density and Power Loss Probs. 28, 29

Index 406