Table of Contents

Preface
Foreword

Chapter 1 Single and Double Hypergeometric Functions

1.1 Introduction 13
1.2 The Integration of the Hypergeometric Differential Equation 15
1.3 Single Hypergeometric Functions of Higher Order 19
1.4 Appell's Double Hypergeometric Functions . 23
1.5 The Kampé de Fériet Function 29
1.6 The Horn Functions 36
1.7 Horn's General Theory of Convergence of Double Hypergeometric Series 37

Chapter 2 The Lauricella Functions

2.1 Introduction and Definitions 41
2.2 Convergence of the Lauricella Series . . 43
2.3 Integral Representations of Euler Type . 48
2.4 Integrals of Laplace Type 49
2.5 Integrals of Barnes Type 51
2.6 Integrals with Respect to Parameters . . 54
2.7 Other Integral Relations 57
2.8 Differential Properties - Addition and Multiplication Theorems 61
2.9 The General Theory of Convergence of Multiple Hypergeometric Series . . . 65
2.10 The Triple Hypergeometric Functions of Lauricella-Saran 66
2.11 An Incomplete $F_D^{(n)}$ Function 70
2.12 Applications 71

Chapter 3 Other Hypergeometric Functions of Several Variables

3.1 The Srivastava Functions H_A, H_B and H_C . 74
3.2 The Pandey Functions 75
3.3 The Quadruple Hypergeometric Functions . 77
3.4 The Multiple Hypergeometric Functions $(k)_{(1)}E_D^{(n)}$, $(k)_{(2)}E_D^{(n)}$ and $(k)_{(1)}E_D^{(n)}$ 89
3.5 The Generalised Horn Functions $(k)H_3^{(n)}$ $(k)H_4^{(n)}$ and $(k)H_2^{(n)}$ 97

3.6 On the Functions $C_n^{(k)}$ and $D_{(n)}^{p,q}$ 104
3.7 Multiple Hypergeometric Functions of Higher Order 106
3.8 Applications 112

Chapter 4 Multiple Hypergeometric Transformations
4.1 Transformations of the Triple and Quadruple Hypergeometric Functions . . . 113
4.2 Euler Integral Transformations of the Lauricella Functions 121
4.3 Transformations Obtainable from the Pochhammer Integrals 124
4.4 Transformations Obtainable from the Laplace Integrals 126
4.5 Special Transformations of ${}_{(1)}^{(1)}E_D^{(n)}$ and $D_{(n)}^{p,q}$ 127
4.6 The Elementary Manipulation of Series . . 131
4.7 Cases of Reducibility 132
4.8 A Generalisation of Bailey's Theorem . . 139
4.9 Certain Transformations Involving Multiple Hypergeometric Functions of Higher Order . 141

Chapter 5 Systems of Partial Differential Equations
5.1 The Partial Differential Systems Associated with the Lauricella Functions 148
5.2 The Broad Nature of the General Integrals . 149
5.3 The System Associated with F_1. (I) . . . 152
5.4 The System Associated with F_1. (II) . . 154
5.5 The Application of Contour Integrals . . 157
5.6 The Integration of the System F_1.(III) . . 160
5.7 The Integration of the System Associated with Φ_2 163
5.8 On the Systems $F_D^{(3)}$ and $F_D^{(4)}$ 166
5.9 The Intehration of the System $F_D^{(n)}$ . . . 171
5.10 On the System $\Phi_2^{(n)}$ 173

Chapter 6 Generating Functions and Recurrence Relations – Analytical Continuation
6.1 Generating Relations of a General Nature . 179
6.2 Applications to Lauricella Polynomials . . 189
6.3 Generating Relations of the Functions ${}_{(1)}^{(k)}E_D^{(n)}$, ${}_{(2)}^{(k)}E_D^{(n)}$ and ${}_{(1)}^{(k)}E_C^{(n)}$ and the Generalised Horn Functions ${}^{(k)}H_3^{(n)}$ and ${}^{(k)}H_4^{(n)}$ 191

6.4 Recurrence Relations for the Lauricella Functions 193
6.5 The Analytical Continuation of the Functions $F_C^{(n)}$, $F_A^{(n)}$ and $F_B^{(n)}$ 197
6.6 The Analytical Continuation of $F_D^{(n)}$ . . 198
6.7 The Analytical Continuation of $F_D^{(3)}$ . . 200
6.8 Certain Analytical Continuation Formulae for $F_D^{(n)}$ when n is unrestricted . . . 210
6.9 Two Formulae of Toscano 215
6.10 The Function R of Carlson 216

Chapter 7 Applications in Statistics
7.1 Special Univariate Distributions . . . 219
7.2 Special Multivariate Distributions . . . 222
7.3 Applications to Bayesian Inference . . . 227
7.4 Characteristic Functions 230
7.5 An Application in the Field of Genetics . 233
7.6 The Distribution of a Sum of Gamma Variates 237
7.7 Distribution of the Ratio 237

Chapter 8 Applications in Physics and Other Fields
8.1 Applications to Ordinary Differential Equations 246
8.2 Asymptotic Solutions of Certain Differential Equations 255
8.3 Approximate Solutions of Differential Equations 260
8.4 A Type of Hyperelliptical Integral . . . 264
8.5 Applications to Quantum Theory and Lie Theory 266
8.6 Applications to Dual Integral Equations . 273
8.7 Integrals of Several Bessel Functions. Applications to the Limiting of Several Sinusoidal Signals with Gaussian Noise . . 276
8.8 Heat Conduction Applications 278
8.9 An Application in the Theory of Elasticity . 281

Appendix A Formulae Relayed to Multiple Hypergeometric Functions 284

Appendix B Two Useful Computer Programs 293

Bibliography 296
Index of Applications 305
Index of Symbols 306
Subject Index 309