

CONTENTS

Foreword	ix
Preface	xi
Preface to the Previous Work	xiii
CHAPTER 1. THE DILOGARITHM	1
1.1. Introduction and Elementary Considerations	1
1.2. Extension to Large Real Values of z	2
1.3. Functional Equations Involving a Single Variable	4
1.4. Numerical Relations	6
1.5. Functional Relations Involving Two Variables	7
1.6. Newman's Functional Equation	11
1.7. Functional Equations Involving Several Variables	16
1.8. Legendre's Chi-function	18
1.9. Some Miscellaneous Results	21
1.10. A Survey of Definitions and Notations	27
1.11. Relations to Other Mathematical Functions	30
1.12. Occurrence in Physical Problems	31
CHAPTER 2. THE INVERSE TANGENT INTEGRAL	38
2.1. Elementary Considerations and Definition	38
2.2. The Inversion Relation	39
2.3. The Duplication Formula	40
2.4. Some Numerical Relations	44
2.5. The Triplication Formula	45
2.6. The Multiplication Formula for Odd Multiples	47
2.7. The Quadruplication Formula	51
2.8. Functional Equations Involving Several Variables	59

CHAPTER 3. THE GENERALIZED INVERSE TANGENT INTEGRAL	68
3.1. Introduction and Elementary Properties	68
3.2. Differentiation with Respect to the Parameter	70
3.3. Formulas Arising from a Change of Variable	74
3.4. Formulas Arising from Inverse Tangent Integrals of Bilinear Argument	78
3.5. Formulas Arising from Inverse Tangent Integrals of Biquadratic Argument	79
3.6. Factorization Theorems	89
3.7. Multiplication Formulas	92
3.8. Derived Relations	93
3.9. Special Values of the Parameter	94
3.10. An Addition Equation Involving Argument and Parameter	96
CHAPTER 4. CLAUSEN'S INTEGRAL	101
4.1. Definition and Elementary Properties	101
4.2. Periodic Properties	102
4.3. The Factorization Theorem	104
4.4. Series Expansions	105
4.5. Integral Relations	106
4.6. Functional Equations	108
4.7. Geometrical Connections	115
CHAPTER 5. THE DILOGARITHM OF COMPLEX ARGUMENT	120
5.1. Resolution into Real and Imaginary Parts	120
5.2. The Factorization Theorem	123
5.3. Special Values of the Argument	124
5.4. Functional Equations Involving a Single Variable	126
5.5. Reduction of $\text{Li}_2(x, \theta)$ for Special Values of θ	131
5.6. Newman's Functional Equation Involving Two Variables	132
5.7. Derived Functional Equations	134
5.8. Consequences of the Duplication Formula	137
5.9. An Addition Formula for the Angular Parameter	139
CHAPTER 6. THE TRILOGARITHM	153
6.1. Introduction and Elementary Considerations	153
6.2. Functional Equations of a Single Variable	154
6.3. Numerical Relations	155
6.4. A Consideration of Some Complex Forms	158
6.5. A Generalization of Clausen's Integral	162
6.6. A Further Consideration of Complex Forms	166
6.7. Functional Equations of Two Variables	172
6.8. A Functional Equation of Newman's Type	185
6.9. Functional Equations Involving Several Variables	187

CHAPTER 7. THE HIGHER-ORDER FUNCTIONS	189
7.1. Introduction and Definitions	189
7.2. The Inversion Equation and Its Consequences	192
7.3. The Factorization Theorems	197
7.4. Associated Integrals	199
7.5. The Associated Clausen Functions	200
7.6. Integral Relations for the Fourth-Order Polylogarithm	202
7.7. Functional Equations for the Fourth-Order Polylogarithm	206
7.8. Functional Equations for the Fifth-Order Polylogarithm	212
7.9. The Log-Sine Integrals	216
7.10. Results from a Contour Integration	228
7.11. Golden-Cut and Related Integrals	232
7.12. Polylogarithms of Nonintegral Order	236
7.13. Higher-Order Polylogarithms	238
CHAPTER 8. INTEGRATION OF FUNCTIONS AND SUMMATION OF SERIES	242
8.1. Reduction of a Class of Algebraic and Logarithmic Expressions	242
8.2. Reduction of Trigonometric Forms	253
8.3. Summation of Series	260
8.4. Integrals from the Higher-Order Functions	269
8.5. Definite Trigonometric Integrals	274
APPENDIX. REFERENCE DATA AND TABLES	281
A.1. Glossary of Notation	281
A.2. List of Selected Formulas	283
A.3. Reference List of Integrals	303
A.4. Tabulated Values	312
Bibliography	349
Suggestions for Further Study	355
Index	357