

## TABLE OF CONTENTS

|  |           |
|--|-----------|
| <b>Preface</b>   | <b>v</b>  |
| <b>Chapter I. Introduction</b>                                   | <b>1</b>  |
| 1. Outline of this book  | 1         |
| 2. Further remarks   | 4         |
| 3. Notation  | 4         |
| <b>Chapter II. Maximum Principles</b>                            | <b>7</b>  |
| 1. The weak maximum principle                                    | 7         |
| 2. The strong maximum principle                                  | 10        |
| 3. A priori estimates  | 14        |
| Notes  | 18        |
| Exercises  | 18        |
| <b>Chapter III. Introduction to the Theory of Weak Solutions</b> | <b>21</b> |
| 1. The theory of weak derivatives                                | 22        |
| 2. The method of continuity                                      | 29        |
| 3. Problems in small balls                                       | 30        |
| 4. Global existence and the Perron process                       | 37        |
| Notes  | 42        |
| Exercises  | 42        |
| <b>Chapter IV. Hölder Estimates</b>                              | <b>45</b> |
| 1. Hölder continuity   | 45        |
| 2. Campanato spaces  | 49        |
| 3. Interior estimates  | 51        |
| 4. Estimates near a flat boundary                                | 61        |
| 5. Regularized distance  | 71        |
| 6. Intermediate Schauder estimates                               | 74        |
| 7. Curved boundaries and nonzero boundary data                   | 76        |
| 8. A special mixed problem                                       | 80        |

|  |            |
|--|------------|
| Notes  | 82         |
| Exercises  | 84         |
| <b>Chapter V. Existence, Uniqueness, and Regularity<br/>of Solutions</b> | <b>87</b>  |
| 1. Uniqueness of solutions   | 87         |
| 2. The Cauchy-Dirichlet problem with bounded coefficients                | 89         |
| 3. The Cauchy-Dirichlet problem with unbounded coefficients              | 94         |
| 4. The oblique derivative problem  | 95         |
| Notes  | 97         |
| Exercises  | 97         |
| <b>Chapter VI. Further Theory of Weak Solutions</b>                      | <b>99</b>  |
| 1. Notation and basic results  | 99         |
| 2. Differentiability of weak derivatives                                 | 107        |
| 3. Sobolev inequalities  | 108        |
| 4. Poincarè's inequality   | 113        |
| 5. Global boundedness  | 115        |
| 6. Local estimates   | 120        |
| 7. Consequences of the local estimates                                   | 128        |
| 8. Boundary estimates  | 132        |
| 9. More Sobolev-type inequalities  | 134        |
| 10. Conormal problems  | 136        |
| 11. Solvability in Hölder spaces   | 140        |
| 12. The parabolic deGiorgi classes                                       | 141        |
| Notes  | 149        |
| Exercises  | 152        |
| <b>Chapter VII. Strong Solutions</b>                                     | <b>155</b> |
| 1. Maximum principles  | 155        |
| 2. Basic results from harmonic analysis                                  | 160        |
| 3. $L^p$ estimates for constant coefficient equations                    | 168        |
| 4. Interior $L^p$ estimates  | 172        |
| 5. Boundary and global estimates   | 173        |
| 6. The oblique derivative problem  | 176        |
| 7. The local maximum principle   | 180        |
| 8. The weak Harnack inequality   | 181        |
| 9. Boundary estimates  | 187        |
| Notes  | 193        |
| Exercises  | 194        |

|  |            |
|--|------------|
| <b>Chapter VIII. Fixed Point Theorems and Their Applications</b>     | <b>197</b> |
| 1. The Schauder fixed point theorem                                  | 199        |
| 2. Applications of the Schauder theorem                              | 200        |
| 3. A theorem of Caristi and its applications                         | 202        |
| Notes  | 211        |
| Exercises  | 212        |
| <b>Chapter IX. Comparison and Maximum Principles</b>                 | <b>213</b> |
| 1. Comparison principles   | 213        |
| 2. Maximum estimates   | 214        |
| 3. Comparison principles for divergence form operators               | 215        |
| 4. Maximum estimates for divergence form operators                   | 216        |
| Notes  | 220        |
| Exercises  | 221        |
| <b>Chapter X. Boundary Gradient Estimates</b>                        | <b>223</b> |
| 1. The boundary gradient estimate in general domains                 | 225        |
| 2. Convex-increasing domains   | 230        |
| 3. The spatial distance function                                     | 233        |
| 4. Curvature conditions  | 236        |
| 5. Nonexistence results  | 240        |
| 6. The case of one space dimension                                   | 246        |
| 7. Continuity estimates  | 247        |
| Notes  | 248        |
| Exercises  | 249        |
| <b>Chapter XI. Global and Local Gradient Bounds</b>                  | <b>251</b> |
| 1. Global estimates for general equations                            | 251        |
| 2. Examples  | 256        |
| 3. Local gradient bounds   | 258        |
| 4. The Sobolev theorem of Michael and Simon                          | 263        |
| 5. Estimates for equations in divergence form                        | 268        |
| 6. The case of one space dimension                                   | 282        |
| 7. A gradient bound for an intermediate situation                    | 285        |
| Notes  | 288        |
| Exercises  | 290        |
| <b>Chapter XII. Hölder Gradient Estimates and Existence Theorems</b> | <b>293</b> |
| 1. Interior estimates for equations in divergence form               | 293        |
| 2. Equations in one space dimension                                  | 294        |

|   |            |
|---|------------|
| 3. Interior estimates for equations in general form                                     | 295        |
| 4. Boundary estimates   | 298        |
| 5. Improved results for nondivergence equations   | 302        |
| 6. Selected existence results   | 306        |
| Notes   | 311        |
| Exercises   | 312        |
| <b>Chapter XIII. The Oblique Derivative Problem for Quasilinear Parabolic Equations</b> | <b>315</b> |
| 1. Maximum estimates  | 316        |
| 2. Gradient estimates for the conormal problem  | 321        |
| 3. Gradient estimates for uniformly parabolic problems in general form                  | 336        |
| 4. The Hölder gradient estimate for the conormal problem                                | 341        |
| 5. Nonlinear boundary conditions with linear equations                                  | 342        |
| 6. The Hölder gradient estimate for quasilinear equations                               | 346        |
| 7. Existence theorems   | 350        |
| Notes   | 352        |
| Exercises   | 354        |
| <b>Chapter XIV. Fully Nonlinear Equations I.</b>  |            |
| <b>Introduction</b>   | <b>357</b> |
| 1. Comparison and maximum principles  | 359        |
| 2. Simple uniformly parabolic equations   | 361        |
| 3. Higher regularity of solutions   | 368        |
| 4. The Cauchy-Dirichlet problem   | 369        |
| 5. Boundary second derivative estimates   | 372        |
| 6. The oblique derivative problem   | 375        |
| 7. The case of one space dimension  | 378        |
| Notes   | 380        |
| Exercises   | 381        |
| <b>Chapter XV. Fully Nonlinear Equations II.</b>  |            |
| <b>Hessian Equations</b>  | <b>383</b> |
| 1. General results for Hessian equations  | 384        |
| 2. Estimates on solutions   | 387        |
| 3. Existence of solutions   | 399        |
| 4. Properties of symmetric polynomials  | 399        |
| 5. The parabolic analog of the Monge-Ampère equation                                    | 406        |

## TABLE OF CONTENTS

xi

|                   |            |
|-------------------|------------|
| Notes             | 413        |
| Exercises         | 417        |
| <b>References</b> | <b>419</b> |
| <b>Index</b>      | <b>437</b> |