Contents

| Introduction | | |
|--|-----|--|
| 0. Some Basic Concepts of Lie Group Representation Theory | 1 | |
| 1. One parameter groups of operators | 1 | |
| 2. Representations of Lie groups, convolution algebras, and Lie algebras | s 9 | |
| 3. Representations of distributions and universal enveloping algebras | 17 | |
| 4. Irreducible representations of Lie groups | 27 | |
| 5. Varieties of Lie groups | 34 | |
| 1. The Heisenberg Group | 42 | |
| 1. Construction of the Heisenberg group \mathbf{H}^n | 42 | |
| 2. Representations of \mathbf{H}^n | 46 | |
| 3. Convolution operators on \mathbf{H}^n and the Weyl calculus | 50 | |
| 4. Automorphisms of \mathbf{H}^n ; the symplectic group | 54 | |
| 5. The Bargmann-Fok representation | 58 | |
| 6. (Sub)Laplacians on \mathbf{H}^n and harmonic oscillators | 61 | |
| 7. Functional calculus for Heisenberg Laplacians and for harmonic | | |
| oscillator Hamiltonians | 67 | |
| 8. The wave equation on the Heisenberg group | 81 | |
| 2. The Unitary Group | 87 | |
| 1. Representation theory for SU(2), SO(3), and some variants | 87 | |
| 2. Representation theory for $U(n)$ | 92 | |
| 3. The subelliptic operator $X_2^2 + X_3^2$ on SU(2) | 98 | |
| 3. Compact Lie Groups | 104 | |
| 1. Weyl orthogonality relations and the Peter-Weyl theorem | 104 | |
| 2. Roots, weights, and the Borel-Weil theorem | 110 | |
| 3. Representations of compact groups on eigenspaces of Laplace | | |
| operators | 110 | |

vi CONTENTS

| 4. | Harmonic Analysis on Spheres 1. The Laplace equation in polar coordinates 2. Classical PDE on spheres 3. Spherical harmonics 4. The subelliptic operators $L_1^2 + L_2^2 + i\alpha L_3$ on S^2 | 128 128 130 133 140 |
|-----|---|---|
| 5. | Induced Representations, Systems of Imprimitivity, and Semidirect Products 1. Induced representations and systems of imprimitivity 2. The Stone-von Neumann theorem 3. Semidirect products 4. The Euclidean group and the Poincaré group | 143 143 146 147 150 |
| 6. | Nilpotent Lie Groups 1. Nilpotent Lie algebras and Lie algebras with dilations 2. Step 2 nilpotent Lie groups 3. Representations of general nilpotent Lie groups | 152 152 154 158 |
| 7. | Harmonic Analysis on Cones 1. Dilations of cones and the $ax+b$ group 2. Spectral representation and functional calculus for the Laplacian on a cone | 163 163 172 |
| 8. | SL(2, R) 1. Introduction to SL(2, R) 2. Classification of irreducible unitary representations 3. The principal series 4. The discrete series 5. The complementary series 6. The spectrum of $L^2(\Gamma\backslash \mathrm{PSL}(2,R))$, in the compact case 7. Harmonic analysis on the Poincaré upper half plane 8. The subelliptic operator $\mathcal{L}_{\alpha} = A^2 + B^2 + \frac{1}{2}i\alpha Z$ on SL(2, R) | 177 177 181 188 193 195 196 199 202 |
| 9. | SL(2, C) and More General Lorentz Groups 1. Introduction to SL(2, C) 2. Representations of SL(2, C) 3. The Lorentz groups SO(n, 1) | 204 204 209 221 |
| 10. | Groups of Conformal Transformations 1. Laplace operators and conformal changes of metric 2. Conformal transformations on \mathbb{R}^n , \mathbb{S}^n , and balls | 226 226 227 |
| 11. | The Symplectic Group and the Metaplectic Group 1. Symplectic vector spaces and the symplectic group 2. Symplectic inner product spaces and compact subgroups of the symplectic group | 235 235 239 |
| | 3. The metaplectic representation | 239 241 |
| | | |

| CONTENTS | vi | ii |
|----------|----|----|
| | | |

| 12. Spinors | 246 |
|--|-------------|
| 1. Clifford algebras and spinors | 246 |
| 2. Spinor bundles and the Dirac operator | 256 |
| 3. Spinors on four-dimensional Riemannian manifolds | 260 |
| 4. Spinors on four-dimensional Lorentz manifolds | 26 4 |
| 13. Semisimple Lie Groups | 268 |
| 1. Introduction to semisimple Lie groups | 268 |
| 2. Some representations of semisimple Lie groups | 279 |
| Appendixes | |
| A. The Fourier transform and tempered distributions | 287 |
| B. The spectral theorem | 292 |
| C. The Radon transform on Euclidean space | 298 |
| D. Analytic vectors, and exponentiation of Lie algebra representations | 300 |
| References | 313 |
| Index | 327 |
| | |