

Contents

Preface	vii
Introduction	xiii
Notations and Conventions	
1 Topological Spaces	1
2 Locally Convex Spaces	2
3 Complexifications	4
4 Unbounded Linear Operators	6
5 General Conventions	7
Chapter I Generators and Interpolation	
1 Generators of Analytic Semigroups	
1.1 Properties of Linear Operators	10
1.2 The Class $\mathcal{H}(E_1, E_0)$	11
1.3 Perturbation Theorems	14
1.4 Spectral Estimates	15
1.5 Compact Perturbations	20
1.6 Matrix Generators	21
2 Interpolation Functors	
2.1 Definitions	24
2.2 Interpolation Inequalities	25
2.3 Retractions	26
2.4 Standard Interpolation Functors	28
2.5 Continuous Injections	30
2.6 Duality Properties	30
2.7 Compactness	31
2.8 Iteration Theorems	31
2.9 Fractional Powers and Interpolation	32
2.10 Semigroups and Interpolation	33
2.11 Admissible Interpolation Functors	35

Chapter II Cauchy Problems and Evolution Operators

1	Linear Cauchy Problems	
1.1	Hölder Spaces	40
1.2	Existence and Regularity Theorems	43
2	Parabolic Evolution Operators	
2.1	Basic Properties	45
2.2	Determining Integral Equations	47
3	Linear Volterra Integral Equations	
3.1	Weakly Singular Kernels	48
3.2	Resolvent Kernels	50
3.3	Singular Gronwall Inequalities	52
4	Existence of Evolution Operators	
4.1	A Class of Parameter Integrals	53
4.2	Semigroup Estimates	55
4.3	Construction of Evolution Operators	57
4.4	The Main Result	63
4.5	Solvability of the Cauchy Problem	66
5	Stability Estimates	
5.1	Estimates for Evolution Operators	68
5.2	Continuity Properties of Mild Solutions	71
5.3	Hölder Estimates	72
5.4	Boundedness of Mild Solutions	74
6	Invariance and Positivity	
6.1	Yosida Approximations	75
6.2	Approximations of Evolution Operators	77
6.3	Invariance	80
6.4	Orderings and Positivity	84

Chapter III Maximal Regularity

1	General Principles	
1.1	Sobolev Spaces	88
1.2	Absolutely Continuous Functions	89
1.3	Generalized Solutions	91
1.4	Trace Spaces	92
1.5	Pairs of Maximal Regularity	94
1.6	Stability	96

2 Maximal Hölder Regularity

2.1 Singular Hölder Spaces 98
 2.2 Semigroup Estimates 102
 2.3 Trace Spaces 106
 2.4 Estimates for K_A 109
 2.5 Maximal Regularity 113
 2.6 Nonautonomous Problems 117

3 Maximal Continuous Regularity

3.1 Necessary Conditions 121
 3.2 Higher Order Interpolation Spaces 123
 3.3 Estimates for K_A 124
 3.4 Maximal Regularity 126

4 Maximal Sobolev Regularity

4.1 Temperate Distributions 128
 4.2 Fourier Transforms and Convolutions 130
 4.3 The Hilbert Transform 135
 4.4 UMD Spaces and Fourier Multipliers 141
 4.5 Properties of UMD Spaces 144
 4.6 Fractional Powers 147
 4.7 Bounded Imaginary Powers 162
 4.8 Perturbation Theorems 168
 4.9 Sums of Closed Operators 173
 4.10 Maximal Regularity 180

Chapter IV Variable Domains

1 Higher Regularity

1.1 Properties of Differentiable Functions 194
 1.2 General Solvability Results for Cauchy Problems 195
 1.3 Estimates for Evolution Operators 198
 1.4 Evolution Operators on Interpolation Spaces 204
 1.5 The Cauchy Problem 207

2 Constant Interpolation Spaces

2.1 Semigroup and Convergence Estimates 211
 2.2 Assumptions and Consequences 214
 2.3 Construction of Evolution Operators 218
 2.4 Estimates for Evolution Operators 227
 2.5 The Cauchy Problem 230
 2.6 Abstract Boundary Value Problems 233

3 Maximal Regularity

3.1	Abstract Initial Boundary Value Problems	242
3.2	Isomorphism Theorems	245

Chapter V Scales of Banach Spaces**1 Banach Scales**

1.1	General Concepts	250
1.2	Power Scales	255
1.3	Extrapolation Spaces	261
1.4	Dual Scales	267
1.5	Interpolation-Extrapolation Scales	275

2 Evolution Equations in Banach Scales

2.1	Semigroups in Interpolation-Extrapolation Scales	286
2.2	Parabolic Evolution Equations in Banach Scales	294
2.3	Duality	297
2.4	Approximation Theorems	300
2.5	Final Value Problems	304
2.6	Weak Solutions and Duality	307
2.7	Positivity	312
2.8	General Evolution Equations	314

Bibliography	321
-------------------------------	-----

List of Symbols	329
----------------------------------	-----

Index	333
------------------------	-----