
Contents

Lattice Dynamical Systems

<i>Shui-Nee Chow</i>	1
1 Spatially Discrete Nonlinear Diffusion Equations	1
1.1 Introduction	1
1.2 Spatially Discrete Models	2
1.3 Gradient Systems	9
1.4 Numerical Methods	13
1.5 Numerical Results	15
2 Dynamics in a Discrete Nagumo Equation – Spatial Chaos	20
2.1 Introduction	20
2.2 Basic Facts	23
2.3 Spatial Chaos	26
2.4 Non-existence of Traveling Waves	31
2.5 Non-existence of Standing Waves	35
3 Pattern Formation and Spatial Chaos in Lattice Dynamical Systems ..	36
3.1 Introduction	36
3.2 Lattice Dynamical Systems	37
3.3 Bifurcation of Checkerboard and Stripe Patterns	41
3.4 Traveling Waves and Prorogation Failure	43
3.5 Homoclinic Points of \mathbb{Z}^D Actions	45
4 Pattern Formation and Spatial Chaos in Spatially Discrete Evolution Equations	46
4.1 Introduction	46
4.2 Existence and Uniqueness of Solutions	47
4.3 Mosaic Solutions: Existence and Stability	53
4.4 Examples of Two-Dimensional Mosaic Solutions	58
5 Spatial Entropy and Spatial Chaos	64
5.1 Spatial Entropy, Spatial Chaos and Pattern Formation	64
5.2 Rigorous Bounds on the Spatial Entropy	72
5.3 One-Dimensional Mosaics	76
6 Synchronization in Lattice Dynamical Systems	80

6.1	Introduction	80
6.2	Statement of Results	81
6.3	Proof of Theorem 6.1	83
6.4	Proof of Theorem 6.2	86
7	Synchronization, Stability and Normal Hyperbolicity	90
7.1	Introduction	90
7.2	Definitions and Results	90
7.3	Synchronization of Fully Coupled Systems	93
7.4	Master-Slave Synchronization and Partially Coupled System	95
	References	99

Totally Bounded Cubic Systems in \mathbb{R}^2

Roberto Conti, Marcello Galeotti 103

1	Finitely Many Singular Points	103
1.1	Totally Bounded Cubic Systems	103
1.2	Singular Points	104
1.3	Annulus of the System	104
1.4	Petals	105
1.5	A Unique Singular Point	105
2	Two Singular Points	106
2.1	Total Boundedness and Reversibility	106
2.2	The Case $D = 0$	108
2.3	Uniqueness of the Singular Points S, O	110
2.4	Conditions of Total Boundedness	111
2.5	Regular Systems	116
2.6	Limit Lines at S	116
2.7	The Cuspidal Case	117
2.8	One Vertical Limit Line at S	119
2.9	Three Limit Lines at S	124
2.10	Petal at S	127
2.11	The Tangential Limit Point O	127
2.12	O Center	133
2.13	O Poincaré Center	133
2.14	O Degenerate Center	136
2.15	O Right Pseudo-center	140
2.16	O Left Pseudo-center	142
2.17	O Poincaré Saddle	146
2.18	O Degenerate Saddle	148
2.19	Two Heteroclines with $index O = 1$	152
2.20	An Invariant Ellipse	157
2.21	An Invariant Ellipse. Two Heteroclines	159
2.22	An Invariant Ellipse. Infinitely Many Heteroclines	163
2.23	Infinitely Many Heteroclines with No Invariant Ellipse	165
3	Limit Cycles	168
3.1	Limit Cycles of Cubic Totally Bounded Systems	168

References 171

Non-Autonomous Differential Equations

Russell Johnson, Francesca Mantellini 173

1 Introduction 173

2 Basic Methods 179

3 Cantor Spectrum for Quasi-Periodic Schroedinger Operators 193

4 Almost Automorphy in Semilinear Parabolic PDEs 206

5 Positive Solutions of the Scalar Curvature Equation 213

References 223

Traveling Waves in Spatially Discrete Dynamical Systems of Diffusive Type

John Mallet-Paret 231

1 Introduction 231

2 Equilibria, Stability, and Patterns 235

3 Traveling Waves in PDE's 239

4 Traveling Waves in Lattice Differential Equations: General Features .. 245

5 An Abstract Approach to Traveling Waves 248

6 Linear Theory of Differential-Difference Equations 252

7 The Global Structure of Traveling Waves in LDE's. 259

8 Pinning 269

9 Crystallographic Pinning 274

10 Stability, Perturbations, and Time-Discretizations 281

11 Exponential Dichotomies for Linear Systems of Mixed Type 286

References 293

Limiting Profiles for Solutions of Differential-Delay Equations

Roger D. Nussbaum 299

1 Background Material: The Fixed Point Index and Measures of Noncompactness 299

2 The Singularly Perturbed Differential-Delay Equation
 $\varepsilon x'(t) = -x(t) + f(x(t-1))$ 310

3 The Differential-Delay Equation $\varepsilon x'(t) = f(x(t), x(t-r(x(t))))$ 324

4 Ω -Theory for Solutions of $\varepsilon x'(t) = f(x(t), x(t-r)), r := r(x(t))$ 332

References 340