

# Contents

<b>Series Preface</b>	<b>vi</b>
<b>Preface</b>	<b>vii</b>
<b>Symbol Index</b>	<b>xiii</b>
<b>1 Special Functions in Applied Mathematics</b>	<b>1</b>
1.1 Variables, Functions, Limits, and Continuity	1
1.2 Why Study Special Functions?	2
1.3 Special Functions and Power Series	3
1.4 The Gamma Function: Another Example from Physics	6
1.5 A Look Ahead	10
Exercises	10
<b>2 Differential Equations and Special Functions</b>	<b>15</b>
2.1 Infinite Series	15
2.2 Analytic Functions	16
2.3 Linear Second-Order Differential Equations	23
2.4 The Hypergeometric Equation	25
2.5 The Simple Pendulum	30
2.6 The Generalized Hypergeometric Function	34
2.7 Vandermonde's Theorem	34
2.8 Leibniz's Theorem	36
Exercises	37
<b>3 The Confluent Hypergeometric Function</b>	<b>41</b>
3.1 The Confluent Hypergeometric Equation	41
3.2 One-Dimensional Harmonic Oscillator	43
Exercises	48
<b>4 Problems in Two Dimensions</b>	<b>53</b>
4.1 Cylindrical Wave Guides	53
4.2 The Vibrating Membrane	64
Exercises	66
<b>5 The Central Force Problem in Quantum Mechanics</b>	<b>69</b>
5.1 Three-Dimensional Schrödinger Equation	69
5.2 Legendre's Equation	72
5.3 Legendre Polynomials and Associated Legendre Functions	75

5.4	Eigenfunctions for a Particle in a Central Field	77
	Exercises	77
<b>6</b>	<b>The Radial Equation for Central Force Fields</b>	<b>81</b>
6.1	Square Well	81
6.2	Spherical Bessel Functions and Related Functions	85
6.3	Free Particle	87
6.4	Coulomb Field	87
6.5	Isotropic Harmonic Oscillator	93
6.6	Special Functions and Hypergeometric Functions	96
	Exercises	98
<b>7</b>	<b>Complex Analysis</b>	<b>99</b>
7.1	Complex Numbers	99
7.2	Analytic Functions of a Complex Variable	101
7.3	Analyticity	110
7.4	Laurent Expansion	118
7.5	Essential Singularities	120
7.6	Branch Points	121
7.7	Analytic Continuation	121
	Exercises	126
<b>8</b>	<b>Applications of Contour Integrals</b>	<b>129</b>
8.1	The Cauchy Residue Theorem	129
8.2	Evaluation of Definite Integrals by Contour Integration	133
	Exercises	147
<b>9</b>	<b>Alternate Forms for Special Functions</b>	<b>155</b>
9.1	The Gamma Function	155
9.2	Bessel Functions	157
9.3	Legendre Polynomials	162
9.4	Hermite Polynomials	165
9.5	Laguerre Polynomials and Associated Laguerre Polynomials	167
	Exercises	167
<b>10</b>	<b>Integral Representations of Special Functions</b>	<b>171</b>
10.1	The Gamma Function	171
10.2	Bessel Functions	173
10.3	Spherical Bessel Functions	178
10.4	Legendre Polynomials	181
10.5	Laguerre Functions	183
10.6	Hermite Polynomials	184
10.7	The Hypergeometric Function	184

Contents	xi
10.8 Asymptotic Expansions	185
10.9 Continuum States in the Coulomb Field	190
Exercises	191
<b>11 Generating Functions and Recursion Formulas</b>	<b>197</b>
11.1 Hermite Polynomials	197
11.2 Laguerre Polynomials	199
11.3 Associated Laguerre Polynomials	201
11.4 Legendre Polynomials	205
11.5 Associated Legendre Functions	208
11.6 A Note on Normalization	209
Exercises	209
<b>12 Orthogonal Functions</b>	<b>213</b>
12.1 Complete Orthonormal Sets of Functions	213
12.2 The Delta Function	214
12.3 Fourier Integrals	218
12.4 Orthogonality and Normalization of Special Functions	223
12.5 Applications of Spherical Harmonics	232
12.6 Sturm–Liouville Theory	236
Exercises	240
<b>Bibliography</b>	<b>245</b>
<b>Index</b>	<b>247</b>