
Contents

List of symbols	<i>page</i>	xi
Introduction		1
1 Preliminaries		15
1.1 Hardy classes		15
1.2 The classes \mathcal{C} and \mathcal{B}		23
1.3 Factorizations		31
1.4 Reproducing kernel spaces		34
1.5 J-unitary and J-contractive matrices		36
2 The fundamental spaces		42
2.1 The spaces \mathcal{L}_n		42
2.2 Calculus in \mathcal{L}_n		53
2.3 Extremal problems in \mathcal{L}_n		58
3 The kernel functions		64
3.1 Christoffel–Darboux relations		64
3.2 Recurrence relations for the kernels		67
3.3 Normalized recursions for the kernels		70
4 Recurrence and second kind functions		74
4.1 Recurrence for the orthogonal functions		74
4.2 Functions of the second kind		82
4.3 General solutions		90
4.4 Continued fractions and three-term recurrence		95
4.5 Points not on the boundary		101

5	Para-orthogonality and quadrature	106
5.1	Interpolatory quadrature	106
5.2	Para-orthogonal functions	108
5.3	Quadrature	112
5.4	The weights	117
5.5	An alternative approach	119
6	Interpolation	121
6.1	Interpolation properties for orthogonal functions	121
6.2	Measures and interpolation	129
6.3	Interpolation properties for the kernels	135
6.4	The interpolation algorithm of Nevanlinna–Pick	140
6.5	Interpolation algorithm for the orthonormal functions	145
7	Density of the rational functions	149
7.1	Density in L_p and H_p	149
7.2	Density in $L_2(\mu)$ and $H_2(\mu)$	155
8	Favard theorems	161
8.1	Orthogonal functions	161
8.2	Kernels	165
9	Convergence	173
9.1	Generalization of the Szegő problem	174
9.2	Further convergence results and asymptotic behavior	181
9.3	Convergence of ϕ_n^*	183
9.4	Equivalence of conditions	191
9.5	Varying measures	192
9.6	Stronger results	196
9.7	Weak convergence	206
9.8	Erdős–Turán class and ratio asymptotics	208
9.9	Root asymptotics	226
9.10	Rates of convergence	233
10	Moment problems	239
10.1	Motivation and formulation of the problem	239
10.2	Nested disks	241
10.3	The moment problem	251

11	The boundary case	257
11.1	Recurrence for points on the boundary	257
11.2	Functions of the second kind	267
11.3	Christoffel–Darboux relation	272
11.4	Green’s formula	277
11.5	Quasi-orthogonal functions	280
11.6	Quadrature formulas	286
11.7	Nested disks	290
11.8	Moment problem	300
11.9	Favard type theorem	307
11.10	Interpolation	319
11.11	Convergence	338
12	Some applications	342
12.1	Linear prediction	343
12.2	Pisarenko modeling problem	356
12.3	Lossless inverse scattering	359
12.4	Network synthesis	369
12.5	H_∞ problems	373
	12.5.1 The standard H_∞ control problem	373
	12.5.2 Hankel operators	379
	12.5.3 Hankel norm approximation	385
	Conclusion	389
	Bibliography	393
	Index	405