
Contents

I	Scalar Wavelets	1
1	Basic Theory	3
1.1	Refinable Functions	3
1.2	Orthogonal MRAs and Wavelets	10
1.3	Wavelet Decomposition	19
1.4	Biorthogonal MRAs and Wavelets	21
1.5	Moments	25
1.6	Approximation Order	28
1.7	Symmetry	32
1.8	Point Values and Normalization	35
2	Practical Computation	39
2.1	Discrete Wavelet Transform	40
2.2	Pre- and Postprocessing	43
2.3	Handling Boundaries	45
2.3.1	Data Extension Approach	46
2.3.2	Matrix Completion Approach	48
2.3.3	Boundary Function Approach	49
2.3.4	Further Comments	51
2.4	Putting It All Together	52
2.5	Modulation Formulation	54
2.6	Polyphase Formulation	56
2.7	Lifting	58
2.8	Calculating Integrals	62
2.8.1	Integrals with Other Refinable Functions	62
2.8.2	Integrals with Polynomials	65
2.8.3	Integrals with General Functions	65
3	Creating Wavelets	69
3.1	Completion Problem	69
3.1.1	Finding Wavelet Functions	69
3.1.2	Finding Dual Scaling Functions	70
3.2	Projection Factors	71
3.3	Techniques for Modifying Wavelets	72
3.4	Techniques for Building Wavelets	73

3.5	Bezout Equation	74
3.6	Daubechies Wavelets	75
3.6.1	Bezout Approach	75
3.6.2	Projection Factor Approach	78
3.7	Coiflets	79
3.7.1	Bezout Approach	79
3.7.2	Projection Factor Approach	81
3.7.3	Generalized Coiflets	82
3.8	Cohen Wavelets	85
3.9	Other Constructions	86
4	Applications	89
4.1	Signal Processing	89
4.1.1	Detection of Frequencies and Discontinuities	90
4.1.2	Signal Compression	90
4.1.3	Denoising	91
4.2	Numerical Analysis	93
4.2.1	Fast Matrix–Vector Multiplication	93
4.2.2	Fast Operator Evaluation	95
4.2.3	Differential and Integral Equations	96
5	Existence and Regularity	97
5.1	Distribution Theory	98
5.2	L^1 -Theory	100
5.3	L^2 -Theory	102
5.3.1	Transition Operator	102
5.3.2	Sobolev Space Estimates	105
5.3.3	Cascade Algorithm	109
5.4	Pointwise Theory	110
5.5	Smoothness and Approximation Order	115
5.6	Stability	116
II	Multiwavelets	121
6	Basic Theory	123
6.1	Refinable Function Vectors	124
6.2	MRAs and Multiwavelets	132
6.2.1	Orthogonal MRAs and Multiwavelets	132
6.2.2	Biorthogonal MRAs and Multiwavelets	137
6.3	Moments	140
6.4	Approximation Order	142
6.5	Point Values and Normalization	147

7	Practical Computation	153
7.1	Discrete Multiwavelet Transform	154
7.2	Pre- and Postprocessing	157
7.2.1	Interpolating Prefilters	158
7.2.2	Quadrature-Based Prefilters	159
7.2.3	Hardin–Roach Prefilters	159
7.2.4	Other Prefilters	161
7.3	Balanced Multiwavelets	161
7.4	Handling Boundaries	163
7.4.1	Data Extension Approach	163
7.4.2	Matrix Completion Approach	164
7.4.3	Boundary Function Approach	164
7.5	Putting It All Together	165
7.6	Modulation Formulation	167
7.7	Polyphase Formulation	169
7.8	Calculating Integrals	171
7.8.1	Integrals with Other Refinable Functions	172
7.8.2	Integrals with Polynomials	174
7.8.3	Integrals with General Functions	175
7.9	Applications	175
7.9.1	Signal Processing	175
7.9.2	Numerical Analysis	175
8	Two-Scale Similarity Transforms	177
8.1	Regular TSTs	177
8.2	Singular TSTs	179
8.3	Multiwavelet TSTs	183
8.4	TSTs and Approximation Order	188
8.5	Symmetry	191
9	Factorizations of Polyphase Matrices	197
9.1	Projection Factors	197
9.1.1	Orthogonal Case	197
9.1.2	Biorthogonal Case	200
9.2	Lifting Steps	203
9.3	Raising Approximation Order by Lifting	206
10	Creating Multiwavelets	209
10.1	Orthogonal Completion	209
10.1.1	Using Projection Factors	209
10.1.2	Householder-Type Approach	211
10.2	Biorthogonal Completion	212
10.3	Other Approaches	214
10.4	Techniques for Modifying Multiwavelets	215
10.5	Techniques for Building Multiwavelets	216

11 Existence and Regularity	219
11.1 Distribution Theory	220
11.2 L^1 -Theory	223
11.3 L^2 -Theory	225
11.3.1 Transition Operator	225
11.3.2 Sobolev Space Estimates	227
11.3.3 Cascade Algorithm	231
11.4 Pointwise Theory	231
11.5 Smoothness and Approximation Order	235
11.6 Stability	235
A Standard Wavelets	239
A.1 Scalar Orthogonal Wavelets	239
A.2 Scalar Biorthogonal Wavelets	240
A.3 Orthogonal Multiwavelets	241
A.4 Biorthogonal Multiwavelets	244
B Mathematical Background	247
B.1 Notational Conventions	247
B.2 Derivatives	247
B.3 Functions and Sequences	247
B.4 Fourier Transform	249
B.5 Laurent Polynomials	250
B.6 Trigonometric Polynomials	250
B.7 Linear Algebra	252
C Computer Resources	255
C.1 Wavelet Internet Resources	255
C.2 Wavelet Software	255
C.3 Multiwavelet Software	257
References	259
Index	270