

CONTENTS

List of Participants	X
Acknowledgements	XIII
Preface.	XVI

PROBLEMS

Chapter 1. ANALYSIS IN FUNCTION SPACES	2
1.1. Uniformly convergent Fourier series	5
1.2. Compactness of absolutely summing operators	7
1.3. When is $\prod_n(X, \ell^2) = L(X, \ell^2)$?	10
1.4c. Local theory of spaces of analytic functions.	14
1.5c. Complemented subspaces of A , H^1 and H^∞	18
1.6. Spaces of Hardy type.	22
1.7. Bases in H^p spaces on the ball	24
1.8. Spaces with the approximation property?	26
1.9. Operator blocks in Banach lattices.	27
1.10c. Isomorphisms and bases.	29
1.11. Isomorphic classification of F-spaces	34
1.12. Weighted spaces of entire functions	38
1.13c. Linear functionals and linear convexity	41
1.14. Supports of analytic functionals.	46
Chapter 2. BANACH ALGEBRAS	48
2.1. The spectral radius in quotient algebra	50
2.2. Extremum problems	51
2.3. Maximum principles for quotient norms in H^∞	53
2.4. Open semigroups in Banach algebras.	55
2.5. Homomorphisms from C^* -algebras	58
2.6. Analyticity in the Gelfand space of multipliers	61
2.7. Homomorphisms of measure algebras	65
2.8. Separation of ideals in group algebras.	68
2.9. Polynomial approximation.	70
2.10. My favourite algebra.	72

2.11c.	Sets of antisymmetry and support sets for $H^\infty + C$	75
2.12.	Subalgebras of the disk algebra.	78
2.13.	Analytic operator families	81
Chapter 3. PROBABILISTIC PROBLEMS		82
3.1c.	Some questions about Hardy functions	85
3.2c.	Analytic problems originating in stationary processes	87
3.3.	Moduli of Hankel operators, Past and Future.	92
3.4.	Strong law of large numbers for stationary processes	98
3.5.	Markov processes and contractions.	101
3.6c.	Existence of measures with given projections	104
3.7c.	An indicator with a spectral gap	106
Chapter 4. OPERATOR THEORY.		108
4.1.	Boundedness of continuum eigenfunctions.	113
4.2.	Scattering theory for Coulomb type problems.	116
4.3.	Polynomial approximation and Hill's equation	121
4.4c.	Zero sets of dissipative operator functions.	124
4.5.	Point spectrum of perturbations of unitary operators	129
4.6.	Spectral analysis of re-expansion operators.	130
4.7.	Non-negative subspaces of \mathcal{J} -dissipative operators	135
4.8.	Perturbation theory and invariant subspaces.	137
4.9c.	Operators and approximation.	140
4.10.	Spectral decompositions and the Carleson condition	144
4.11.	Similarity problem and the singular spectrum	147
4.12c.	Analytic operator-valued functions	152
4.13.	Invariant subspaces of C_{10} -contractions.	155
4.14.	Titchmarsh's theorem for vector functions.	158
4.15.	Operator functions and spectral measures of isometries	160
4.16.	\mathcal{J} -inner matrix-functions.	164
4.17.	Extremal multiplicative representations.	169
4.18.	Factorization of operators on $L^2(a, b)$	172
4.19.	An infinite product of special matrices.	177
4.20.	Factorization of operator functions	180
4.21.	When are differentiable functions differentiable?	184
4.22c.	Are multiplication and shift approximable?	189
4.23.	Extremal similarities.	197
4.24.	Estimates of functions of Hilbert space operators.	199
4.25.	Estimates of operator polynomials on \mathcal{P}_p	205
4.26.	2x2-Matsaev conjecture	209
4.27.	Diminishing of spectrum under an extension	210

4.28.	The decomposition of Riesz operators	211
4.29.	Free invertibility of Fredholm operators	212
4.30.	Indices of an operator matrix and its determinant. . .	214
4.31.	Compact operators with power-like s-numbers.	217
4.32.	Perturbation of spectrum for normal operators.	219
4.33.	Perturbation of continuous spectrum.	223
4.34.	Almost-normal operators modulo \mathcal{I}_p	227
4.35.	Hyponormal operators and spectral absolute continuity	231
4.36.	Operators, analytic negligibility, and capacities. . .	234
4.37.	Generalized differentiations and semidiagonality . . .	238
4.38.	What is a finite operator?	240
4.39.	Spectra of endomorphisms of a Banach algebra	244
4.40.	Composition of integration and substitution	249
Chapter 5. HANKEL AND TOEPLITZ OPERATORS		251
5.1c.	Approximation by elements of $H^\infty + C$	254
5.2.	Quasinilpotent Hankel operators.	259
5.3.	Hankel operators on Bergman spaces	262
5.4c.	Similarity for Toeplitz operators.	264
5.5.	Iterates of Toeplitz operators	269
5.6.	Localization of Toeplitz operators	271
5.7.	Toeplitz operators on the Bergman space.	274
5.8.	Vectorial Toeplitz operators on Hardy spaces	276
5.9.	Factorization of almost periodic matrices.	279
5.10.	Toeplitz operators in several variables.	283
5.11.	Around Szegő limit theorems.	285
5.12.	Moments, Toeplitz matrices and statistical physics. .	289
5.13.	Reduction method for Toeplitz operators.	293
5.14.	Elliptizität und Projektionsverfahren	298
5.15.	Defect numbers of Riemann boundary value problem . . .	303
5.16.	Poincaré-Bertrand operators in Banach algebras	306
Chapter 6. SINGULAR INTEGRALS, BMO, H^p		308
6.1c.	The Cauchy integral and related operators.	310
6.2c.	Classes of domains and Cauchy type integrals	313
6.3.	Bilinear singular integrals.	317
6.4.	Weighted norm inequalities	318
6.5.	Weak type substitute for Riesz projections on tori . .	322
6.6.	The norm of the analytic projection.	325
6.7.	Is this operator invertible?	328
6.8.	BMO-norm and operator norm	329

6.9c. Problems concerning H^∞ and BMO	330
6.10c. Two conjectures by Albert Baernstein.	333
6.11c. Blaschke products in \mathcal{B}_0	337
6.12. Algebras contained within H^∞	339
6.13. Analytic functions in W_2^1	341
6.14. Subalgebras of $L^\infty(\mathbb{T}^2)$ containing $H^\infty(\mathbb{T}^2)$	342
6.15. Inner functions with derivative in H^p , $0 < p < 1$	343
6.16. Equivalent norms in H^p	345
6.17. A definition of H^p	346
6.18. Hardy classes and Riemann surfaces.	347
6.19. Interpolating Blaschke products	351
Chapter 7. SPECTRAL ANALYSIS AND SYNTHESIS	353
7.1. Holomorphic functions with limited growth	357
7.2. $\bar{\partial}$ -equation and localization of submodules	361
7.3c. Invariant subspaces and differential equations.	364
7.4. Local description of closed submodules.	367
7.5. Spectral synthesis for entire functions	372
7.6c. Spectral synthesis for differential operators	374
7.7. Two problems on the spectral synthesis.	378
7.8c. Cyclic vectors in spaces of analytic functions.	382
7.9. Weak invertibility and factorization.	386
7.10c. Weakly invertible elements in Bergman spaces.	390
7.11. Invariant subspaces of the backward shift	393
7.12. Divisibility problems in $A(\mathbb{D})$ and $H^\infty(\mathbb{D})$	396
7.13. A refinement of the corona theorem	399
7.14. Invariant subspaces of the shift	401
7.15. Blaschke products and ideals in C_A^∞	403
7.16. Closed ideals in the analytic Gevrey class.	407
7.17. Completeness of translates in a weighted space	409
7.18. Problems of harmonic analysis in weighted spaces.	414
7.19c. A closure problem for functions on \mathbb{R}_+	417
7.20. Translates of functions of two variables.	421
7.21. Algebra and ideal generation.	422
7.22. Harmonic synthesis and compositions	426
7.23c. Deux problèmes sur les séries trigonométriques.	429
Chapter 8. APPROXIMATION AND CAPACITIES	431
8.1c. Spectral synthesis in Sobolev spaces.	435
8.2. Approximation by smooth functions in Sobolev spaces	438
8.3c. Splitting in H^2 -spaces	439

8.4.	Trigonometric approximation in $L^2(\mathbb{R}, d\Delta)$	447
8.5.	Decomposition of approximable functions	449
8.6.	Approximation and quasiconformal continuation	451
8.7.	Tangential approximation.	453
8.8.	Integrability of the derivative of a conformal mapping	457
8.9.	Weighted polynomial approximation	461
8.10.	Approximation in the mean by harmonic functions	466
8.11.	Rational approximation of analytic functions.	471
8.12c.	Padé approximation in several variables	475
8.13.	Badly-approximable functions.	480
8.14.	Exotic Jordan arcs in \mathbb{C}^N	483
8.15.	Removable sets for bounded analytic functions	485
8.16.	On Painlevé null sets	491
8.17.	Analytic capacity and rational approximation	495
8.18.	On sets of analytic capacity zero	498
8.19.	Estimates of analytic capacity.	502
8.20c.	Regularität für elliptische Gleichungen	507
8.21.	Exceptional sets for Besov spaces	515
8.22.	Complex interpolation between Sobolev spaces.	519
Chapter 9. UNIQUENESS, MOMENTS, NORMALITY		520
9.1c.	Representations of analytic functions	522
9.2.	Moment problem questions.	529
9.3c.	Uniqueness and finite Dirichlet integral.	531
9.4.	Stationary functions, uncertainty, Jordan operators	536
9.5.	Problem in the theory of functions.	541
9.6.	Peak sets for Lipschitz classes	544
9.7.	A problem by R.Kaufman.	547
9.8c.	Quasi-analyticity and differential operators.	548
9.9c.	Local operators on Fourier transforms	552
9.10.	Density of exponentials on plane arcs	555
9.11.	When is $\int \log f > -\infty$?	557
9.12.	An alternative for analytic Carleman classes.	558
9.13.	On a uniqueness theorem in \mathbb{C}^n	561
Chapter 10. INTERPOLATION, BASES, MULTIPLIERS		563
10.1c.	Interpolation by entire functions	566
10.2.	Bases of reproducing kernels and exponentials	569
10.3c.	Multiplicative properties of ℓ_A^p	572
10.4.	Free interpolation in regular classes	575
10.5.	Traces of H^∞ -functions on hyperplanes.	577

10.6.	Representations by exponential series	579
10.7.	Restrictions of the Lipschitz spaces	583
10.8.	Multipliers, interpolation, and $\Lambda(p)$ -sets	586
Chapter 11. ENTIRE AND SUBHARMONIC FUNCTIONS		589
11.1c.	The inverse problem of best approximation	591
11.2.	Derivatives of unbounded analytic functions	595
11.3.	Exceptional values of various kinds	597
11.4.	Valiron exceptional values	599
11.5c.	Preservation of the completely regular growth	600
11.6c.	Zero-sets of sine-type functions.	605
11.7.	An extremal problem for subharmonic functions	609
11.8c.	A problem on exact majorants.	611
11.9.	Entire functions of Laguerre-Pólya class	614
11.10.	Cluster sets and a problem of A.F.Leont'ev.	617
Chapter 12. \mathbb{C}^n		619
12.1.	Polynomially convex hulls	620
12.2c.	Positive pluriharmonic functions	623
12.3.	Proper mappings of classical domains.	625
12.4.	On biholomorphy of holomorphic mappings	629
Chapter 13. MISCELLANEOUS PROBLEMS		631
13.1.	Banach algebras and almost periodicity.	632
13.2.	Support points of univalent functions	636
13.3.	More problems by Albert Baernstein.	638
13.4.	Some extension problems	639
13.5.	Partition of singularities.	641
13.6.	Rearrangement-invariant hulls of sets	642
13.7.	Norms and extremals of convolution operators.	646
13.8c.	Algebraic equations with Banach algebra coefficients	652
13.9.	Holomorphic mappings and algebraic functions.	657
13.10.	Singular points of plane algebraic curves	662

SOLUTIONS

S.1c.	Absolutely summing operators	665
S.2c.	Golubev series and analyticity on continua.	670
S.3c.	The vanishing interior of the spectrum.	674
S.4c.	Uniqueness for mean periodic functions.	677
S.5c.	The Cauchy integral on Lipschitz graphs	679

S.6c.	Sets of uniqueness for QC	682
S.7c.	Another problem by R.Kaufman	684
S.8c.	Rational functions with given ramifications.	686
S.9c.	Asymptotic behaviour of entire functions	688
S.10c.	The inner function problem in balls.	691
S.11c.	Homogeneous measures on subsets of \mathbb{R}^n	698
Subject Index	700
Author Index	709
Standard notation.	721