

CONTENTS

Preface	ix
Conventions, notations	xix

FIRST PART

INTRODUCTION	3
CHAPTER 1 IDEALIZATION	
1.1 Set theory, new predicate	7
1.2 Axiomatic: beginning	10
1.3 Comments	12
1.4 First applications of (I)	13
1.5 More comments suggested by the examples	14
1.6 Natural integers	17
1.7 Power sets	18
1.8 External sets	18
1.9 Exercises	19
CHAPTER 2 STANDARDIZATION AND TRANSFER	
2.1 Axiomatic: end	21
2.2 Consequences	22
2.3 An example	25
2.4 Standard finite sets	25
2.5 Functions and graphs	27
2.6 Relativization	29
2.7 Final comments on axiomatics	30
2.8 Exercises	31
CHAPTER 3 REAL NUMBERS AND NUMERICAL FUNCTIONS	
3.1 Basic concepts	35
3.2 Standard part	38
3.3 Generalization to topological spaces	42

3.4 Sequences	44
3.5 Exercises	47
CHAPTER 4 CONTINUITY	
4.1 S-continuity	51
4.2 Examples showing the difference between continuity and S-continuity	52
4.3 Relations between continuity and S-continuity	54
4.4 Uniform continuity	56
4.5 Theorems on continuous functions	58
4.6 Exercises	61
CHAPTER 5 DIFFERENTIABILITY	
5.1 Differentiable functions	63
5.2 Theorems for differentiable functions	66
5.3 Strictly differentiable functions	68
5.4 Higher derivatives	70
5.5 Finite differences and derivatives	71
5.6 Exercises	73
CHAPTER 6 INTEGRATION	
6.1 Method	75
6.2 Definite integral	77
6.3 Strict standard part of a function	80
6.4 Exercises	81
SECOND PART	
CHAPTER 7 INVARIANT MEANS	
7.1 Defining properties	85
7.2 Existence of invariant means	86
7.3 A result	88
7.4 More comments	88
7.5 Exercises	91
CHAPTER 8 APPROXIMATION OF FUNCTIONS	
8.1 Dirac functions	93
8.2 Periodic functions and trigonometric polynomials	95
8.3 Bernstein polynomial approximation	98
8.4 Approximation in quadratic mean	103
8.5 Exercises	105
CHAPTER 9 DIFFERENTIAL EQUATIONS	
9.1 Review of some classical notions	107
9.2 Existence theorem	109
9.3 An example	112

CHAPTER 10 PERTURBATION OF A GREEN FUNCTION	
10.1 Green function.....	115
10.2 Nonstandard perturbation	116
10.3 Origin of the problem	119
CHAPTER 11 INVARIANT SUBSPACES PROBLEM	
11.1 Situation of the result	123
11.2 Preliminary results	124
11.3 Proof of the Bernstein–Robinson Theorem	126
11.4 Comments.....	130
INDICATIONS FOR EXERCISES.....	133
SOLUTIONS OF EXERCISES	137
BIBLIOGRAPHY.....	153
INDEX.....	155
BASIC PRINCIPLES OF NSA	157
1ST AXIOMS FOR NSA	159