

Contents

Preface	xii
Lectures on L-functions, Converse Theorems, and Functoriality for $GL(n)$	
James W. Cogdell	
Preface	3
Lecture 1. Modular Forms and Their L -functions	5
1. Examples	6
2. Growth estimates on cusp forms	7
3. The L -function of a cusp form	8
4. The Euler product	10
5. References	12
Lecture 2. Automorphic Forms	13
1. Automorphic forms on GL_2	13
2. Automorphic forms on GL_n	16
3. Smooth automorphic forms	17
4. L^2 -automorphic forms	18
5. Cusp forms	18
6. References	19
Lecture 3. Automorphic Representations	21
1. (K -finite) automorphic representations	21
2. Smooth automorphic representations	24
3. L^2 -automorphic representations	25
4. Cuspidal representations	25
5. Connections with classical forms	26
6. References	27
Lecture 4. Fourier Expansions and Multiplicity One Theorems	29
1. The Fourier expansion of a cusp form	29
2. Whittaker models	31
3. Multiplicity one for GL_n	33
4. Strong multiplicity ones for GL_n	34
5. References	35

Lecture 5. Eulerian Integral Representations	37
1. $GL_2 \times GL_1$	37
2. $GL_n \times GL_m$ with $m < n$	38
3. $GL_n \times GL_n$	41
4. Summary	43
5. References	43
 Lecture 6. Local L -functions: The Non-Archimedean Case	45
1. Whittaker functions	45
2. The local L -function ($m < n$)	46
3. The local functional equation	48
4. The conductor of π	49
5. Multiplicativity and stability of γ -factors	49
6. References	50
 Lecture 7. The Unramified Calculation	51
1. Unramified representations	52
2. Unramified Whittaker functions	53
3. Calculating the integral	55
4. References	57
 Lecture 8. Local L -functions: The Archimedean Case	59
1. The arithmetic Langlands classification	59
2. The L -functions	59
3. The integrals ($m < n$)	61
4. Is the L -factor correct?	62
5. References	64
 Lecture 9. Global L -functions	65
1. Convergence	65
2. Meromorphic continuation	66
3. Poles of L -functions	67
4. The global functional equation	67
5. Boundedness in vertical strips	68
6. Summary	69
7. Strong Multiplicity One revisited	69
8. Generalized Strong Multiplicity One	70
9. References	70
 Lecture 10. Converse Theorems	73
1. Converse Theorems for GL_n	73
2. Inverting the integral representation	74
3. Proof of Theorem 10.1 (i)	77
4. Proof of Theorem 10.1 (ii)	77
5. Theorem 10.2 and beyond	78
6. A useful variant	79
7. Conjectures	79

8. References	80
Lecture 11. Functoriality	81
1. The Weil-Deligne group	81
2. The dual group	82
3. The local Langlands conjecture	82
4. Local functoriality	83
5. Global functoriality	83
6. Functoriality and the Converse Theorem	84
7. References	85
Lecture 12. Functoriality for the Classical Groups	87
1. The results	87
2. Construction of a candidate lift	88
3. Analytic properties of L -functions	90
4. Apply the Converse Theorem	90
5. References	90
Lecture 13. Functoriality for the Classical Groups, II	91
1. Functoriality	91
2. Descent	92
3. Bounds towards Ramanujan	94
4. The local converse theorem	94
5. Further applications	95
6. References	96
Automorphic L-functions	
Henry H. Kim	
Introduction	99
Chapter 1. Chevalley Groups and their Properties	101
1. Algebraic groups	101
2. Roots and coroots	103
3. Classification of root systems	104
4. Construction of Chevalley groups: simply connected type	107
5. Structure of parabolic subgroups	108
Chapter 2. Cuspidal Representations	113
Chapter 3. L -groups and Automorphic L -functions	115
Chapter 4. Induced Representations	119
1. Harish-Chandra homomorphisms	119
2. Induced representations: F local	121
3. Intertwining operators for $I(s, \pi)$	122
4. Digression on admissible representations	123
5. Induced representations: F global	126

6. Induced representations as holomorphic fiber bundles	126
Chapter 5. Eisenstein Series and Constant Terms	129
1. Definition of Eisenstein series	129
2. Constant terms	130
3. Pseudo-Eisenstein series	132
Chapter 6. L -functions in the Constant Terms	137
List of L -functions via Langlands-Shahidi method	143
Chapter 7. Meromorphic Continuation of L -functions	145
Chapter 8. Generic Representations and their Whittaker Models	147
1. General case	147
2. Whittaker models for induced representations	149
Chapter 9. Local Coefficients and Non-constant Terms	153
1. Non-constant terms of Eisenstein series	153
2. Local coefficients and crude functional equation	158
Chapter 10. Local Langlands Correspondence	161
Chapter 11. Local L -functions and Functional Equations	165
1. Definition of local L -functions	169
2. Properties of local L -functions; supercuspidal representations	170
Chapter 12. Normalization of Intertwining Operators	171
1. π is supercuspidal	171
2. π is tempered, generic	171
3. π is non-tempered, generic	172
4. Application to reducibility criterion	175
Chapter 13. Holomorphy and Bounded in Vertical Strips	177
1. Holomorphy of L -functions	177
2. Boundedness in vertical strips of L -functions	177
Chapter 14. Langlands Functoriality Conjecture	181
Chapter 15. Converse Theorem of Cogdell and Piatetski-Shapiro	183
Chapter 16. Functoriality of the Symmetric Cube	187
1. Weak Ramanujan property	187
2. Functoriality of the symmetric square	187
3. Functoriality of the tensor product of $GL_2 \times GL_3$	188
4. Functoriality of the symmetric cube	190
Chapter 17. Functoriality of the Symmetric Fourth	193
1. Functoriality of the exterior square	193

2. Functoriality of the symmetric fourth	194
Bibliography	199
Applications of Symmetric Power L-functions	
M. Ram Murty	
Preface	205
Lecture 1. The Sato-Tate Conjecture	207
1. Introduction	207
2. Uniform distribution	208
3. Wiener-Ikehara Tauberian theorem	209
4. Weyl's theorem for compact groups	210
Lecture 2. Maass Wave Forms	213
1. Maass forms of weight zero	213
2. Maass forms with weight	214
3. Eisenstein series	214
4. Upper bound for Fourier coefficients and eigenvalue estimators	216
Lecture 3. The Rankin-Selberg Method	219
1. Eisenstein series and non-vanishing of $\zeta(s)$ on $\Re(s) = 1$	219
2. Explicit construction of Maass cusp forms	221
3. The Rankin-Selberg L -function	222
4. Rankin-Selberg L -functions for GL_n	225
Lecture 4. Oscillations of Fourier Coefficients of Cusp Forms	227
1. Preliminaries	227
2. Rankin's theorem	228
3. A review of symmetric power L -functions	230
4. Proof of Theorem 4.1	232
Lecture 5. Poincaré Series	237
1. Poincaré series for $SL_2(\mathbb{Z})$	237
2. Fourier coefficients and Kloosterman sums	239
3. The Kloosterman-Selberg zeta function	242
Lecture 6. Kloosterman Sums and Selberg's Conjecture	243
1. Petersson's formula	243
2. Selberg's theorem	244
3. The Selberg-Linnik conjecture	245
Lecture 7. Refined Estimates for Fourier Coefficients of Cusp Forms	247
1. Sieve theory and Kloosterman sums	247
2. Gauss sums and hyper-Kloosterman sum	248
3. The Duke-Iwaniec method	248

Lecture 8. Twisting and Averaging of L -series	253
1. Selberg conjectures for GL_n	253
2. Ramanujan conjecture for Gl_n	254
3. The method of averaging L -functions	255
Lecture 9. The Kim-Sarnak Theorem	257
1. Preliminaries	257
2. Rankin-Selberg theory	258
3. An application of the Duke-Iwaniec method	259
Lecture 10. Introduction to Artin L -functions	265
1. Hecke L -functions	265
2. Artin L -functions	266
3. Automorphic induction and Artin's conjecture	268
Lecture 11. Zeros and Poles of Artin L -functions	271
1. The Heilbronn character	271
2. The fundamental inequality	272
3. Rankin-Selberg property for Galois representations	273
Lecture 12. The Langlands-Tunnell Theorem	275
1. Review of some group theory	275
2. Some representation theory	276
3. An application of the Deligne-Serre theory	277
4. The general case	277
5. Sarnak's theorem	278
Bibliography	281