## **Contents**

| Preface to the English Edition                             |    |
|--|----|
| Preface to the Japanese Edition                            | x  |
| Chapter 1. Fundamental Properties of Holomorphic Functions | 1  |
| Summary  |    |
| §1. Continuous functions, measures, and distributions      | 1  |
| §2. Holomorphic functions                                  | 3  |
| §3. Power series and Reinhardt domains                     | ě  |
| §4. Linear topological spaces of holomorphic functions     | 8  |
| §5. Germs of holomorphic functions                         | 11 |
| §6. Runge open sets  | 13 |
| §7. The Fourier-Borel transformation                       | 14 |
| §8. Entire functions of exponential type                   | 16 |
| Chapter 2. Analytic Functionals of One Variable            | 23 |
| Summary  | 23 |
| §1. The Cauchy-Hilbert transformation                      | 24 |
| §2. The Runge theorems                                     | 30 |
| §3. The Mittag-Leffler theorem                             | 32 |
| §4. A representation of analytic functionals               | 35 |
| §5. The Fourier-Laplace transform of an entire function of |    |
| exponential type   | 36 |
| §6. Convolution  | 38 |
| Chapter 3. Hyperfunctions of One Variable                  | 41 |
| Summary  | 41 |
| §1. Definition of hyperfunctions                           | 42 |
| §2. Locality of hyperfunctions                             | 45 |
| §3. Various operations                                     | 47 |
| §4. $\delta$ -function and Y-function                      | 49 |
| §5. Power functions  | 53 |
| §6. Singular spectrum                                      | 55 |
| §7. Relation with local analytic functionals               | 58 |
| §8. Ordinary differential equations                        | 61 |
| §9. Distributions and hyperfunctions                       | 64 |

V

contents

| Chapter 4. Cohomology Groups with Coefficients in a Sheaf  | 69     |
|--|--------|
| Summary  | 69     |
| §1. Sheaf  | 69     |
| §2. Presheaf   | 74     |
| §3. Cohomology groups with coefficients in a sheaf   | 79     |
| §4. Relative cohomology groups   | 85     |
| §5. The cohomology group of a covering   | 87     |
| §6. The theory of sheaves over paracompact spaces  | 93     |
| §7. The derived sheaf of relative cohomology groups  | 97     |
| §8. Pure codimensionality and the flabby dimension of a sheaf  | 99     |
| §9. Theorems on pure codimensionality  | 104    |
| Chapter 5. Cohomology Groups with Coefficients in $\mathscr O$   | 109    |
| Summary  | 109    |
| §1. Domain of holomorphy (review)  | 109    |
| §2. A method of calculation of relative cohomology groups with   |        |
| coefficients in @  | 115    |
| §3. Vanishing of relative cohomology groups  | 119    |
| Chapter 6. Analytic Functionals of Several Variables   | 129    |
| Summary  | 129    |
| §1. Integral representation of holomorphic functions   | 129    |
| §2. Linearly convex compact sets   | 133    |
| §3. Dual space of $\mathscr{O}(K)$   | 134    |
| §4. The Laplace-Martineau transform of entire functions of   |        |
| exponential type   | 141    |
| §5. The Martineau-Harvey duality theorem   | 146    |
| Chapter 7. Hyperfunctions of Several Variables   | 155    |
| Summary  | 155    |
| §1. Definition of hyperfunctions and their fundamental properties  | 155    |
| §2. Hyperfunctions as a class of holomorphic functions   | 158    |
| §3. Hyperfunctions with compact support  | 163    |
| §4. The boundary value of a holomorphic function   | 165    |
| §5. Injectivity of the boundary value operator $\mathbf{b}_{\Gamma}$   | 170    |
| Chapter 8. Microfunctions  | 177    |
| Summary  | 177    |
| §1. The decomposition of singularities of hyperfunctions of one variable   | 170    |
| §2. Conormal sphere bundle   | 178    |
| §3. Vanishing of the relative cohomology group   | 181    |
| §4. The Mayer-Vietoris theorem   | 183    |
| §5. Definition of microfunction  | 188    |
| §6. Relation of hyperfunctions and microfunctions  | 190    |
| §7. Microfunctions and the analyticity of hyperfunctions   | 197    |
| TA MANAGEMENT OF THE THE ANALYTICAL OF THE TRANSPORT OF THE TAXABLE OF TAXABLE OF THE TAXABLE OF TA | // 1/1 |

| CONTENTS | vii |
|----------|-----|
|----------|-----|

| Chapter 9. Development of Hyperfunction Theory        | 211 |
|---|-----|
| Summary   | 211 |
| §1. Hyperfunctions on a real analytic manifold        | 211 |
| §2. Edge-of-the-Wedge theorem                         | 214 |
| §3. Micro-analyticity of hyperfunctions               | 218 |
| §4. Operations on hyperfunctions                      | 222 |
| §5. Sato's fundamental theorem                        | 228 |
| §6. Lorentz invariant hyperfunctions                  | 232 |
| Appendix A. Linear Topological Spaces                 | 239 |
| Summary   | 239 |
| §1. Linear topological spaces                         | 239 |
| §2. Limit of linear spaces                            | 243 |
| §3. Limit of linear topological spaces                | 244 |
| §4. FS spaces   | 246 |
| §5. DFS spaces  | 249 |
| §6. Duality of FS spaces and DFS spaces               | 253 |
| Appendix B. Rudiments of Homological Algebra          | 257 |
| Summary   | 257 |
| §1. Exact sequences                                   | 257 |
| §2. Cohomology group of a complex                     | 258 |
| Appendix C. Bibliographical Notes                     | 261 |
| Measure theory and function theory                    | 261 |
| Distribution theory and functional analysis           | 261 |
| Early papers on hyperfunctions                        | 262 |
| Sheaf theory and homological algebra                  | 262 |
| Analytic functionals                                  | 263 |
| Microfunctions  | 263 |
| Theoretical physics and the Edge-of-the-Wedge theorem | 263 |
| More recent literature                                | 264 |
| Bibliography  | 265 |
| Subject Index   | 269 |