CONTENTS

Page

CHAPTER I

ABSTRACT HILBERT SPACE AND ITS REALIZATIONS

§ 1. The Concept of Space 1
§ 2. Abstract Hilbert Space 2
§ 3. Abstract Unitary Spaces 16
§ 4. Linear Manifolds in Hilbert Space 18
§ 5. Realizations of Abstract Hilbert Space 23

CHAPTER II

TRANSFORMATIONS IN HILBERT SPACE

§ 1. Linear Transformations 33
§ 2. Symmetric Transformations 49
§ 3. Bounded Linear Transformations 53
§ 4. Projections 70
§ 5. Isometric and Unitary Transformations 76
§ 6. Unitary Invariance 83

CHAPTER III

EXAMPLES OF LINEAR TRANSFORMATIONS

§ 1. Infinite Matrices 86
§ 2. Integral Operators 98
§ 3. Differential Operators 112
§ 4. Operators of Other Types 124

CHAPTER IV

RESOLVENTS, SPECTRA, REDUCIBILITY

§ 1. The Fundamental Problems 125
§ 2. Resolvents and Spectra 128
§ 3. Reducibility 150

CHAPTER V

SELF-ADJOINT TRANSFORMATIONS

§ 1. Analytical Methods 155
§ 2. Analytical Representation of the Resolvent 165
§ 3. The Reducibility of the Resolvent 172

Page

§ 4. The Analytical Representation of a Self-Adjoint Transformation 180
§ 5. The Spectrum of a Self-Adjoint Transformation .. 184

CHAPTER VI

THE OPERATIONAL CALCULUS

§ 1. The Radon-Stieltjes Integral 198
§ 2. The Operational Calculus 221

CHAPTER VII

THE UNITARY EQUIVALENCE OF SELF-ADJOINT TRANSFORMATIONS

§ 1. Preparatory Theorems 242
§ 2. Unitary Equivalence 247
§ 3. Self-Adjoint Transformations with Simple Spectra 275
§ 4. The Reducibility of Self-Adjoint Transformations . 288
§ 5. Reduction to Principal Axes 294

CHAPTER VIII

GENERAL TYPES OF LINEAR TRANSFORMATIONS

§ 1. Permutability 299
§ 2. Unitary Transformations 302
§ 3. Normal Transformations 311
§ 4. A Theorem on Factorization 331

CHAPTER IX

SYMMETRIC TRANSFORMATIONS

§ 1. The General Theory 334
§ 2. Real Transformations 357
§ 3. Approximation Theorems 365

CHAPTER X

APPLICATIONS

§ 1. Integral Operators 397
§ 2. Ordinary Differential Operators of the First Order 424
§ 3. Ordinary Differential Operators of the Second Order 448
§ 4. Jacobi Matrices and Allied Topics 530

Index 615