

CONTENTS

Preface

vii

Chapter 1. First- and Second-order Differential Equations

1.1. Introduction	1
1.2. The First-order Linear Differential Equation	2
1.3. Fundamental Inequality	3
1.4. Second-order Linear Differential Equations	5
1.5. Inhomogeneous Equation	7
1.6. Lagrange Variation of Parameters	8
1.7. Two-point Boundary Value Problem	10
1.8. Connection with Calculus of Variations	11
1.9. Green's Functions	12
1.10. Riccati Equation	14
1.11. The Cauchy-Schwarz Inequality	16
1.12. Perturbation and Stability Theory	18
1.13. A Counter-example	20
1.14. $\int_{-\infty}^{\infty} f(t) dt < \infty$	21
1.15. $\int_{-\infty}^{\infty} f'(t) dt < \infty$	22
1.16. Asymptotic Behavior	23
1.17. The Equation $u'' - (1 + f(t))u = 0$	24
1.18. More Refined Asymptotic Behavior	26
1.19. $\int_{-\infty}^{\infty} f^2 dt < \infty$	27
1.20. The Second Solution	29
1.21. The Liouville Transformation	30
1.22. Elimination of Middle Term	31
1.23. The WKB Approximation	33
1.24. The One-dimensional Schrödinger Equation	33
1.25. $u'' + (1 + f(t))u = 0$; Asymptotic Behavior	33
1.26. Asymptotic Series	35
1.27. The Equation $u' = p(u, t)/q(u, t)$	37
1.28. Monotonicity of Rational Functions of u and t	38
1.29. Asymptotic Behavior of Solutions of $u' = p(u, t)/q(u, t)$	39
Miscellaneous Exercises	42
Bibliography and Comments	51

Chapter 2. Matrix Theory

2.1. Introduction	54
2.2. Determinantal Solution	55

2.3. Elimination	58
2.4. Ill-conditioned Systems	59
2.5. The Importance of Notation	60
2.6. Vector Notation	60
2.7. Norm of a Vector	61
2.8. Vector Inner Product	61
2.9. Matrix Notation	63
2.10. Noncommutativity	64
2.11. The Adjoint, or Transpose, Matrix	65
2.12. The Inverse Matrix	65
2.13. Matrix Norm	67
2.14. Relative Invariants	68
2.15. Constrained Minimization	71
2.16. Symmetric Matrices	72
2.17. Quadratic Forms	74
2.18. Multiple Characteristic Roots	75
2.19. Maximization and Minimization of Quadratic Forms	76
2.20. Min-Max Characterization of the λ_k	77
2.21. Positive Definite Matrices	79
2.22. Determinantal Criteria	81
2.23. Representation for A^{-1}	82
2.24. Canonical Representation for Arbitrary A	82
2.25. Perturbation of Characteristic Frequencies	84
2.26. Separation and Reduction of Dimensionality	85
2.27. Ill-conditioned Matrices and Tychonov Regularization	86
2.28. Self-consistent Approach	88
2.29. Positive Matrices	88
2.30. Variational Characterization of $\lambda(A)$	89
2.31. Proof of Minimum Property	91
2.32. Equivalent Definition of $\lambda(A)$	92
Miscellaneous Exercises	94
Bibliography and Comments	101

Chapter 3. Matrices and Linear Differential Equations

3.1. Introduction	104
3.2. Vector-Matrix Calculus	104
3.3. Existence and Uniqueness of Solution	105
3.4. The Matrix Exponential	107
3.5. Commutators	108
3.6. Inhomogeneous Equation	110
3.7. The Euler Solution	111
3.8. Stability of Solution	113
3.9. Linear Differential Equation with Variable Coefficients	114
3.10. Linear Inhomogeneous Equation	116
3.11. Adjoint Equation	118
3.12. The Equation $X' = AX + XB$	118
3.13. Periodic Matrices: the Floquet Representation	120
3.14. Calculus of Variations	121
3.15. Two-point Boundary Condition	122

3.16. Green's Functions	123
3.17. The Matrix Riccati Equation	123
3.18. Kronecker Products and Sums	124
3.19. $AX + XB = C$	125
3.20. Random Difference Systems	127
Miscellaneous Exercises	127
Bibliography and Comments	131

Chapter 4. Stability Theory and Related Questions

4.1. Introduction	134
4.2. Dini-Hukuhara Theorem—I	135
4.3. Dini-Hukuhara Theorem—II	138
4.4. Inverse Theorems of Perron	140
4.5. Existence and Uniqueness of Solution	140
4.6. Poincaré–Lyapunov Stability Theory	142
4.7. Proof of Theorem	143
4.8. Asymptotic Behavior	146
4.9. The Function $\varphi(c)$	148
4.10. More Refined Asymptotic Behavior	149
4.11. Analysis of Method of Successive Approximations	150
4.12. Fixed-point Methods	152
4.13. Time-dependent Equations over Finite Intervals	152
4.14. Alternative Norm	155
4.15. Perturbation Techniques	156
4.16. Second Method of Lyapunov	157
4.17. Solution of Linear Systems	157
4.18. Origins of Two-point Boundary Value Problems	158
4.19. Stability Theorem for Two-point Boundary Value Problem	159
4.20. Asymptotic Behavior	160
4.21. Numerical Aspects of Linear Two-point Boundary Value Problems	161
4.22. Difference Methods	163
4.23. Difference Equations	165
4.24. Proof of Stability	165
4.25. Analysis of Stability Proof	166
4.26. The General Concept of Stability	168
4.27. Irregular Stability Problems	168
4.28. The Emden–Fowler–Fermi–Thomas Equation	170
Miscellaneous Exercises	171
Bibliography and Comments	182

Chapter 5. The Bubnov–Galerkin Method

5.1. Introduction	187
5.2. Example of the Bubnov–Galerkin Method	188
5.3. Validity of Method	189
5.4. Discussion	190
5.5. The General Approach	190
5.6. Two Nonlinear Differential Equations	192

5.7. The Nonlinear Spring	193
5.8. Alternate Average	196
5.9. Straightforward Perturbation	196
5.10. A "Tucking-in" Technique	198
5.11. The Van der Pol Equation	198
5.12. Two-point Boundary Value Problems	200
5.13. The Linear Equation $L(u) = g$	200
5.14. Method of Moments	202
5.15. Nonlinear Case	202
5.16. Newton-Raphson Method	204
5.17. Multidimensional Newton-Raphson	207
5.18. Choice of Initial Approximation	208
5.19. Nonlinear Extrapolation and Acceleration of Convergence	210
5.20. Alternatives to Newton-Raphson	211
5.21. Lagrange Expansion	212
5.22. Method of Moments Applied to Partial Differential Equations	214
Miscellaneous Exercises	215
Bibliography and Comments	222

Chapter 6. Differential Approximation

6.1. Introduction	225
6.2. Differential Approximation	225
6.3. Linear Differential Operators	226
6.4. Computational Aspects—I	226
6.5. Computational Aspects—II	227
6.6. Degree of Approximation	228
6.7. Orthogonal Polynomials	229
6.8. Improving the Approximation	231
6.9. Extension of Classical Approximation Theory	231
6.10. Riccati Approximation	232
6.11. Transcendentally-transcendent Functions	233
6.12. Application to Renewal Equation	233
6.13. An Example	236
6.14. Differential-Difference Equations	238
6.15. An Example	239
6.16. Functional-Differential Equations	240
6.17. Reduction of Storage in Successive Approximations	242
6.18. Approximation by Exponentials	242
6.19. Mean-square Approximation	242
6.20. Validity of the Method	243
6.21. A Bootstrap Method	244
6.22. The Nonlinear Spring	244
6.23. The Van der Pol Equation	246
6.24. Self-consistent Techniques	248
6.25. The Riccati Equation	248
6.26. Higher-order Approximation	250
6.27. Mean-square Approximation—Periodic Solutions	251
Miscellaneous Exercises	253
Bibliography and Comments	255

Chapter 7. The Rayleigh–Ritz Method

7.1. Introduction	259
7.2. The Euler Equation	259
7.3. The Euler Equation and the Variational Problem	260
7.4. Quadratic Functionals: Scalar Case	261
7.5. Positive Definiteness for Small T	263
7.6. Discussion	264
7.7. The Rayleigh–Ritz Method	265
7.8. Validity of the Method	265
7.9. Monotone Behavior and Convergence	267
7.10. Estimation of $\ u - v\ $ in Terms of $J(v) - J(u)$	268
7.11. Convergence of Coefficients	269
7.12. Alternate Estimate	270
7.13. Successive Approximations	271
7.14. Determination of the Coefficients	272
7.15. Multidimensional Case	273
7.16. Reduction of Dimension	274
7.17. Minimization of Inequalities	275
7.18. Extension to Quadratic Functionals	277
7.19. Linear Integral Equations	279
7.20. Nonlinear Euler Equation	280
7.21. Existence and Uniqueness	281
7.22. Minimizing Property	282
7.23. Convexity and Uniqueness	282
7.24. Implied Boundedness	283
7.25. Lack of Existence of Minimum	284
7.26. Functional Analysis	284
7.27. The Euler Equation and Haar's Device	286
7.28. Discussion	287
7.29. Successive Approximations	288
7.30. Lagrange Multiplier	288
7.31. A Formal Solution Is a Valid Solution	289
7.32. Raising the Price Diminishes the Demand	289
7.33. The Courant Parameter	290
7.34. Control Theory	291
Miscellaneous Exercises	291
Bibliography and Comments	301

Chapter 8. Sturm–Liouville Theory

8.1. Equations Involving Parameters	304
8.2. Stationary Values	305
8.3. Characteristic Values and Functions	306
8.4. Properties of Characteristic Values and Functions	307
8.5. Generalized Fourier Expansion	312
8.6. Discussion	313
8.7. Rigorous Formulation of Variational Problem	314
8.8. Rayleigh–Ritz Method	315
8.9. Intermediate Problem of Weinstein	316

8.10. Transplantation	316
8.11. Positive Definiteness of Quadratic Functionals	317
8.12. Finite Difference Approximations	318
8.13. Monotonicity	319
8.14. Positive Kernels	320
Miscellaneous Exercises	322
Bibliography and Comment	329
<i>Author Index</i>	331
<i>Subject Index</i>	337