

|| CONTENTS

INTRODUCTION	1
I THE ABSTRACT APPROACH TO LINEAR PROBLEMS	4
I.1 Abstract Linear Spaces	5
I.2 Examples of Linear Spaces	10
I.3 Linear Operators	13
I.4 Linear Operators in Finite-Dimensional Spaces	18
I.5 Other Examples of Linear Operators	21
I.6 Direct Sums and Quotient Spaces	28
I.7 Linear Functionals	31
I.8 Linear Functionals in Finite-Dimensional Spaces	35
I.9 Zorn's Lemma	37
I.10 Extension Theorems for Linear Operators	38
I.11 Hamel Bases	41
I.12 The Transpose of a Linear Operator	44
I.13 Annihilators, Ranges, and Null Spaces	45
I.14 Conclusions	49
II TOPOLOGICAL LINEAR SPACES	51
II.1 Normed Linear Spaces	52
II.2 Examples of Normed Linear Spaces	56
II.3 Finite-Dimensional Normed Linear Spaces	62
II.4 Banach Spaces	66
II.5 Quotient Spaces	71
II.6 Inner-Product Spaces	73
II.7 Hilbert Space	86
II.8 Examples of Complete Orthonormal Sets	91
II.9 Topological Linear Spaces	94
II.10 Convex Sets	100
II.11 Locally Convex Spaces	105
II.12 Minkowski Functionals	111
II.13 Metrizable Topological Linear Spaces	115
III LINEAR FUNCTIONALS AND WEAK TOPOLOGIES	121
III.1 Linear Varieties and Hyperplanes	122
III.2 The Hahn-Banach Theorem	125
III.3 The Conjugate of a Normed Linear Space	134

III.4	The Second Conjugate Space	139
III.5	Some Representations of Linear Functionals	141
III.6	Weak Topologies for Linear Spaces	156
III.7	Polar Sets and Annihilators	160
III.8	Equicontinuity and \mathfrak{S} -topologies	165
III.9	The Principle of Uniform Boundedness	169
III.10	Weak Topologies for Normed Linear Spaces	172
III.11	The Krein–Milman Theorem	181
IV	GENERAL THEOREMS ON LINEAR OPERATORS	188
IV.1	Spaces of Linear Operators	189
IV.2	Integral Equations of the Second Kind	196
IV.3	\mathcal{L}^2 Kernels	201
IV.4	Differential Equations and Integral Equations	205
IV.5	Closed Linear Operators	208
IV.6	Some Representations of Bounded Linear Operators	219
IV.7	The M. Riesz Convexity Theorem	224
IV.8	Conjugates of Linear Operators	226
IV.9	Theorems About Continuous Inverses	234
IV.10	The States of an Operator and Its Conjugate	237
IV.11	Adjoint Operators	242
IV.12	Projections	246
IV.13	Fredholm Operators	253
V	SPECTRAL ANALYSIS OF LINEAR OPERATORS	264
V.1	Analytic Vector-Valued Functions	265
V.2	The Resolvent Operator	272
V.3	The Spectrum of a Bounded Linear Operator	277
V.4	Subdivisions of the Spectrum	282
V.5	Reducibility	287
V.6	The Ascent and Descent of an Operator	289
V.7	Compact Operators	293
V.8	An Operational Calculus	309
V.9	Spectral Sets. The Spectral Mapping Theorem	320
V.10	Isolated Points of the Spectrum	328
V.11	Operators with a Rational Resolvent	336
VI	SPECTRAL ANALYSIS IN HILBERT SPACE	341
VI.1	Bilinear and Quadratic Forms	342
VI.2	Symmetric Operators	345
VI.3	Normal and Self-adjoint Operators	349
VI.4	Compact Symmetric Operators	353
VI.5	Symmetric Operators with Compact Resolvent	361
VI.6	The Spectral Theorem for Bounded Self-adjoint Operators	363
VI.7	Unitary Operators	374
VI.8	Unbounded Self-adjoint Operators	380

VII BANACH ALGEBRAS	386
VII.1 Examples of Banach Algebras	387
VII.2 Spectral Theory in a Banach Algebra	393
VII.3 Ideals and Homomorphisms	400
VII.4 Commutative Banach Algebras	404
VII.5 Applications and Extensions of the Gelfand Theory	415
VII.6 B^* -algebras	426
VII.7 The Spectral Theorem for a Normal Operator	430
BIBLIOGRAPHY	445
LIST OF SPECIAL SYMBOLS	455
INDEX	459