

CONTENTS

	PAGE
PREFACE	vii
BIBLIOGRAPHY	xvi
1. INTERPOLATION	
1. Introduction	1
2. The Taylor expansion	2
3. The finite Taylor series with the remainder term	3
4. Interpolation by polynomials	5
5. The remainder of Lagrangian interpolation formula	6
6. Equidistant interpolation	8
7. Local and global interpolation	11
8. Interpolation by central differences	13
9. Interpolation around the midpoint of the range	16
10. The Laguerre polynomials	17
11. Binomial expansions	21
12. The decisive integral transform	24
13. Binomial expansions of the hypergeometric type	26
14. Recurrence relations	27
15. The Laplace transform	29
16. The Stirling expansion	32
17. Operations with the Stirling functions	34
18. An integral transform of the Fourier type	35
19. Recurrence relations associated with the Stirling series	37
20. Interpolation of the Fourier transform	40
21. The general integral transform associated with the Stirling series	42
22. Interpolation of the Bessel functions	45
2. HARMONIC ANALYSIS	
1. Introduction	49
2. The Fourier series for differentiable functions	50
3. The remainder of the finite Fourier expansion	53
4. Functions of higher differentiability	56
5. An alternative method of estimation	58
6. The Gibbs oscillations of the finite Fourier series	60
7. The method of the Green's function	66
8. Non-differentiable functions. Dirac's delta function	68

	PAGE
9. Smoothing of the Gibbs oscillations by Fejér's method	71
10. The remainder of the arithmetic mean method	72
11. Differentiation of the Fourier series	74
12. The method of the sigma factors	75
13. Local smoothing by integration	76
14. Smoothing of the Gibbs oscillations by the sigma method	78
15. Expansion of the delta function	80
16. The triangular pulse	81
17. Extension of the class of expandable functions	83
18. Asymptotic relations for the sigma factors	84
19. The method of trigonometric interpolation	89
20. Error bounds for the trigonometric interpolation method	91
21. Relation between equidistant trigonometric and polynomial interpolations	93
22. The Fourier series in curve fitting	98
3. MATRIX CALCULUS	
1. Introduction	100
2. Rectangular matrices	102
3. The basic rules of matrix calculus	103
4. Principal axis transformation of a symmetric matrix	106
5. Decomposition of a symmetric matrix	111
6. Self-adjoint systems	113
7. Arbitrary $n \times m$ systems	115
8. Solvability of the general $n \times m$ system	118
9. The fundamental decomposition theorem	120
10. The natural inverse of a matrix	124
11. General analysis of linear systems	127
12. Error analysis of linear systems	129
13. Classification of linear systems	134
14. Solution of incomplete systems	139
15. Over-determined systems	141
16. The method of orthogonalisation	142
17. The use of over-determined systems	144
18. The method of successive orthogonalisation	148
19. The bilinear identity	152
20. Minimum property of the smallest eigenvalue	158
4. THE FUNCTION SPACE	
1. Introduction	163
2. The viewpoint of pure and applied mathematics	164
3. The language of geometry	165
4. Metrical spaces of infinitely many dimensions	166
5. The function as a vector	167
6. The differential operator as a matrix	170

	PAGE
7. The length of a vector	173
8. The scalar product of two vectors	175
9. The closeness of the algebraic approximation	175
10. The adjoint operator	179
11. The bilinear identity	181
12. The extended Green's identity	182
13. The adjoint boundary conditions	184
14. Incomplete systems	187
15. Over-determined systems	190
16. Compatibility under inhomogeneous boundary conditions	192
17. Green's identity in the realm of partial differential operators	195
18. The fundamental field operations of vector analysis	198
19. Solution of incomplete systems	201
5. THE GREEN'S FUNCTION	
1. Introduction	206
2. The role of the adjoint equation	207
3. The role of Green's identity	208
4. The delta function $\delta(x, \xi)$	208
5. The existence of the Green's function	211
6. Inhomogeneous boundary conditions	217
7. The Green's vector	220
8. Self-adjoint systems	225
9. The calculus of variations	229
10. The canonical equations of Hamilton	230
11. The Hamiltonisation of partial operators	237
12. The reciprocity theorem	239
13. Self-adjoint problems. Symmetry of the Green's function	241
14. Reciprocity of the Green's vector	241
15. The superposition principle of linear operators	244
16. The Green's function in the realm of ordinary differential operators	247
17. The change of boundary conditions	255
18. The remainder of the Taylor series	256
19. The remainder of the Lagrangian interpolation formula	258
20. Lagrangian interpolation with double points	263
21. Construction of the Green's vector	266
22. The constrained Green's function	270
23. Legendre's differential equation	275
24. Inhomogeneous boundary conditions	278
25. The method of over-determination	281
26. Orthogonal expansions	286
27. The bilinear expansion	291
28. Hermitian problems	299
29. The completion of linear operators	308

	PAGE
6. COMMUNICATION PROBLEMS	
1. Introduction	315
2. The step function and related functions	315
3. The step function response and higher order responses	320
4. The input-output relation of a galvanometer	323
5. The fidelity problem of the galvanometer response	325
6. Fidelity damping	327
7. The error of the galvanometer recording	328
8. The input-output relation of linear communication devices	330
9. Frequency analysis	334
10. The Laplace transform	336
11. The memory time	337
12. Steady state analysis of music and speech	339
13. Transient analysis of noise phenomena	342
7. STURM-LIOUVILLE PROBLEMS	
1. Introduction	348
2. Differential equations of fundamental significance	349
3. The weighted Green's identity	352
4. Second order operators in self-adjoint form	356
5. Transformation of the dependent variable	359
6. The Green's function of the general second order differential equation	364
7. Normalisation of second order problems	368
8. Riccati's differential equation	370
9. Periodic solutions	371
10. Approximate solution of a differential equation of second order	374
11. The joining of regions	376
12. Bessel functions and the hypergeometric series	378
13. Asymptotic properties of $J_p(z)$ in the complex domain	380
14. Asymptotic expression of $J_p(x)$ for large values of x	382
15. Behaviour of $J_p(z)$ along the imaginary axis	384
16. The Bessel functions of the order $\frac{1}{2}$	385
17. Jump conditions for the transition "exponential-periodic"	387
18. Jump conditions for the transition "periodic-exponential"	388
19. Amplitude and phase in the periodic domain	389
20. Eigenvalue problems	390
21. Hermite's differential equation	391
22. Bessel's differential equation	394
23. The substitute functions in the transitory range	400
24. Tabulation of the four substitute functions	404
25. Increased accuracy in the transition domain	405
26. Eigensolutions reducible to the hypergeometric series	409
27. The ultraspherical polynomials	410

	PAGE
28. The Legendre polynomials	412
29. The Laguerre polynomials	418
30. The exact amplitude equation	420
31. Sturm-Liouville problems and the calculus of variations	425
8. BOUNDARY VALUE PROBLEMS	
1. Introduction	432
2. Inhomogeneous boundary conditions	435
3. The method of the "separation of variables"	438
4. The potential equation of the plane	439
5. The potential equation in three dimensions	448
6. Vibration problems	454
7. The problem of the vibrating string	456
8. The analytical nature of hyperbolic differential operators	464
9. The heat flow equation	469
10. Minimum problems with constraints	472
11. Integral equations in the service of boundary value problems	476
12. The conservation laws of mechanics	479
13. Unconventional boundary value problems	486
14. The eigenvalue $\lambda = 0$ as a limit point	487
15. Variational motivation of the parasitic spectrum	494
16. Examples for the parasitic spectrum	498
17. Physical boundary conditions	504
18. A universal approach to the theory of boundary value problems	508
9. NUMERICAL SOLUTION OF TRAJECTORY PROBLEMS	
1. Introduction	512
2. Differential equations in normal form	513
3. Trajectory problems	514
4. Local expansions	515
5. The method of undetermined coefficients	517
6. Lagrangian interpolation in terms of double points	520
7. Extrapolations of maximum efficiency	521
8. Extrapolations of minimum round-off	521
9. Estimation of the truncation error	524
10. End-point extrapolation	526
11. Mid-point interpolations	527
12. The problem of starting values	529
13. The accumulation of truncation errors	531
14. The method of Gaussian quadrature	534
15. Global integration by Chebyshev polynomials	536
16. Numerical aspects of the method of global integration	540
17. The method of global correction	546
Appendix	551
Index	555