

Contents

<i>Preface</i>	v
<i>Introduction</i>	vii
<i>Contents of Other Volumes</i>	xv

I: PRELIMINARIES

1. <i>Sets and functions</i>	1
2. <i>Metric and normed linear spaces</i>	3
<i>Appendix Lim sup and lim inf</i>	11
3. <i>The Lebesgue integral</i>	12
4. <i>Abstract measure theory</i>	19
5. <i>Two convergence arguments</i>	26
6. <i>Equicontinuity</i>	28
<i>Notes</i>	31
<i>Problems</i>	32

II: HILBERT SPACES

1. <i>The geometry of Hilbert space</i>	36
2. <i>The Riesz lemma</i>	41
3. <i>Orthonormal bases</i>	44
4. <i>Tensor products of Hilbert spaces</i>	49
5. <i>Ergodic theory: an introduction</i>	54
<i>Notes</i>	60
<i>Problems</i>	63

III: BANACH SPACES

1. <i>Definition and examples</i>	67
2. <i>Duals and double duals</i>	72
3. <i>The Hahn–Banach theorem</i>	75
4. <i>Operations on Banach spaces</i>	78
5. <i>The Baire category theorem and its consequences</i>	79
Notes	84
Problems	86

IV: TOPOLOGICAL SPACES

1. <i>General notions</i>	90
2. <i>Nets and convergence</i>	95
3. <i>Compactness</i>	97
Appendix <i>The Stone–Weierstrass theorem</i>	103
4. <i>Measure theory on compact spaces</i>	104
5. <i>Weak topologies on Banach spaces</i>	111
Appendix <i>Weak and strong measurability</i>	115
Notes	117
Problems	119

V: LOCALLY CONVEX SPACES

1. <i>General properties</i>	124
2. <i>Fréchet spaces</i>	131
3. <i>Functions of rapid decrease and the tempered distributions</i>	133
Appendix <i>The N-representation for \mathcal{S} and \mathcal{S}'</i>	141
4. <i>Inductive limits: generalized functions and weak solutions of partial differential equations</i>	145
5. <i>Fixed point theorems</i>	150
6. <i>Applications of fixed point theorems</i>	153
7. <i>Topologies on locally convex spaces: duality theory and the strong dual topology</i>	162
Appendix <i>Polars and the Mackey–Arens theorem</i>	167
Notes	169
Problems	173

VI: BOUNDED OPERATORS

1. <i>Topologies on bounded operators</i>	182
2. <i>Adjoins</i>	185
3. <i>The spectrum</i>	188
4. <i>Positive operators and the polar decomposition</i>	195
5. <i>Compact operators</i>	198
6. <i>The trace class and Hilbert–Schmidt ideals</i>	206
<i>Notes</i>	213
<i>Problems</i>	216

VII: THE SPECTRAL THEOREM

1. <i>The continuous functional calculus</i>	221
2. <i>The spectral measures</i>	224
3. <i>Spectral projections</i>	234
4. <i>Ergodic theory revisited: Koopmanism</i>	237
<i>Notes</i>	243
<i>Problems</i>	245

VIII: UNBOUNDED OPERATORS

1. <i>Domains, graphs, adjoints, and spectrum</i>	249
2. <i>Symmetric and self-adjoint operators: the basic criterion for self-adjointness</i>	255
3. <i>The spectral theorem</i>	259
4. <i>Stone's theorem</i>	264
5. <i>Formal manipulation is a touchy business: Nelson's example</i>	270
6. <i>Quadratic forms</i>	276
7. <i>Convergence of unbounded operators</i>	283
8. <i>The Trotter product formula</i>	295
9. <i>The polar decomposition for closed operators</i>	297
10. <i>Tensor products</i>	298
11. <i>Three mathematical problems in quantum mechanics</i>	302
<i>Notes</i>	305
<i>Problems</i>	312

THE FOURIER TRANSFORM

1. The Fourier transform on $\mathcal{S}(\mathbb{R}^n)$ and $\mathcal{S}'(\mathbb{R}^n)$, convolutions	318
2. The range of the Fourier transform: Classical spaces	326
3. The range of the Fourier transform: Analyticity	332
Notes	338
Problems	339

SUPPLEMENTARY MATERIAL

II.2. Applications of the Riesz lemma	344
III.1. Basic properties of L^p spaces	348
IV.3. Proof of Tychonoff's theorem	351
IV.4. The Riesz–Markov theorem for $X = [0, 1]$	353
IV.5. Minimization of functionals	354
V.5. Proofs of some theorems in nonlinear functional analysis	363
VI.5. Applications of compact operators	368
VIII.7. Monotone convergence for forms	372
VIII.8. More on the Trotter product formula	377
Uses of the maximum principle	382
Notes	385
Problems	387

List of Symbols	393
Index	395