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Chapter V. Some Properties of Bases in Concrete Banach Spaces

Appendix I. Bases in General (not Necessarily Separable) Banach Spaces

Appendix II. Bases in Topological Linear Spaces