CONTENTS

INTRODUCTION..... 1

STANDARD DEFINITIONS, NOTATIONS AND CONVENTIONS..... 6

PART I. SEQUENCE SPACES

1. SCHAUDER BASES

a. Existence of bases, the approximation property..... 9

b. Equivalence of bases, uniqueness, stability..... 13

c. Unconditional bases..... 15

d. Subspaces of spaces with unconditional bases..... 17

e. Block bases..... 19

f. Operators on sequence spaces..... 22

2. THE SPACES ℓ_p and c_0

a. Block bases of the unit vector basis; characterizations of ℓ_p and c_0..... 24

b. Block bases of the unit vector basis - applications..... 30

c. Uniqueness of unconditional bases, Grothendieck's inequality..... 32

d. Special properties of ℓ_1 and ℓ_∞..... 37

e. Dvoretzky's theorem and a related open problem..... 42

f. Subspaces of ℓ_p and c_0..... 44

3. SYMMETRIC BASES

a. Examples of symmetric bases..... 53

b. Block bases with constant coefficients and their applications..... 58

c. Embedding spaces with unconditional bases into spaces with symmetric bases..... 62

d. Pełczynski's universal space, uniqueness of symmetric bases..... 64

4. ORLICZ SEQUENCE SPACES

a. Subspaces of Orlicz sequence spaces; especially subspaces with symmetric bases..... 67

b. Subspaces of Orlicz sequence spaces isomorphic to ℓ_p-spaces..... 72

c. Quotient spaces and complemented subspaces..... 74

d. Minimal and universal Orlicz functions..... 78

e. Uniqueness and non-uniqueness of symmetric bases in Orlicz sequence spaces..... 81

f. Examples of Orlicz sequence spaces..... 84

g. Lorentz sequence spaces..... 90

PART II. FUNCTION SPACES

1. BANACH LATTICES

a. Introduction, definitions and examples..... 93

b. Boolean algebras of projections..... 96

c. *Reflexive and weakly sequentially complete lattices*..... 99

d. *Rearrangement invariant spaces and conditional expectations*..... 101

e. *Uniqueness of representation as a lattice*..... 107

2. LATTICE CHARACTERIZATIONS OF SOME CLASSICAL SPACES

a. *Abstract* L *and* M - *spaces*..... 110

b. *Joint characterizations of* L_p *and* M - *spaces*..... 114

c. *Ultraproducts of Banach spaces and their application to* L_p*-spaces*..... 119

3. *THE* L_p*-SPACES*

a. *Isometric classification of* L_p*-spaces and contractive projections*..... 124

b. *Uniformly convex spaces*..... 127

c. *Subspaces of* $L_p(0,1)$; $1 \leq p < \infty$, *negative definite functions*..... 132

d. *Classical interpolation theorems*..... 140

e. *The trigonometric system,* $\Lambda(p)$*-sets*..... 142

f. *The Haar system*..... 146

g. *Subspaces of* L_p; $1 \leq p < 2$, *and especially subspaces with a symmetric structure*..... 147

h. *Orlicz function spaces*..... 149

i. *Remarks on other results on* L_p*-spaces*..... 152

4. *THE* C(K)-*SPACES AND PREDUALS OF* L_1-*SPACES*

a. Isometric classification of the C(K)-*spaces*..... 153

b. $C_\sigma(K)$-*spaces and* G-*spaces*..... 155

c. Preduals of L_1-*spaces and their extension properties*..... 157

d. P_1-*spaces*..... 160

e. Further classes of preduals of L_1-*spaces; characterization of* C(K)-*spaces among them*..... 162

f. Matrix representation of preduals of L_1-*spaces; the Gurarii space*..... 165

g. Simultaneous extensions and selections..... 169

h. Averaging operators..... 172

i. Isomorphic classification of C(K)-*spaces; first part: Milutin's theorem*..... 174

j. Isomorphic classification of C(K)-*spaces; second part: countable or non-metrizable* K..... 177

k. Weakly compact operators; the Dunford-Pettis property..... 181

l. Projections in C(K)-*spaces*..... 184

m. Separably injective Banach spaces..... 187

n. Injective Banach spaces..... 190

5. *THE* $\mathcal{L}_p$-*SPACES*

a. Introduction, local reflexivity..... 195

b. The definition of $\mathcal{L}_p$-*spaces and their basic properties*... 197

c. Existence of bases in $\mathcal{L}_p$-*spaces*..... 203

d. Examples of $\mathcal{L}_1$-*spaces*..... 209

e. Examples of $\mathcal{L}_p$-*spaces;* $1 < p < \infty$, $p \neq 2$..... 214

f. *Characterization of* $\mathcal{L}_p$*-spaces;* p = 1, 2 *and* ∞, *by extension and lifting properties of operators*..... 219

g. *Further characterization of* $\mathcal{L}_p$*-spaces; Banach spaces having sufficiently many Boolean algebras of projections*... 224

R E F E R E N C E S..... 231