

# CONTENTS

## CHAPTER I

### CONTRACTIONS AND THEIR DILATIONS

1. Unilateral shifts. Wold decomposition . . . . .	1
2. Bilateral shifts . . . . .	4
3. Contractions. Invariant vectors and canonical decomposition . . . . .	6
4. Isometric and unitary dilations of a contraction . . . . .	10
5. Matrix construction of the unitary dilation . . . . .	16
6. Commutative systems of isometries and contractions . . . . .	19
7. Positive definite functions on a group . . . . .	24
8. Some applications . . . . .	28
9. Regular unitary dilations of commutative systems of contractions . . . . .	32
10. Another method to construct isometric dilations. An example concerning systems of differential equations . . . . .	39
11. Unitary $\varrho$ -dilations . . . . .	45
<i>Notes</i> . . . . .	51

## CHAPTER II

### GEOMETRICAL AND SPECTRAL PROPERTIES OF THE MINIMAL UNITARY DILATION

1. Structure of the space of the minimal unitary dilation. . . . .	57
2. Structure of the space of the minimal isometric dilation. Dilation of commutants . . . . .	61
3. The residual part of the minimal unitary dilation. Quasi-affinities and quasi-similarity ties . . . . .	68
4. A classification of contractions. Canonical triangulations . . . . .	72
5. Invariant subspaces and their behaviour with respect to quasi-similarity . . . . .	76
6. Spectral relations . . . . .	81
7. Spectral multiplicity . . . . .	86
8. Similarity of the operators of class $C_\theta$ to contractions . . . . .	92
<i>Notes</i> . . . . .	95

## CHAPTER III

### FUNCTIONAL CALCULUS: BOUNDED FUNCTIONS

1. Hardy classes. Inner and outer functions . . . . .	99
2. Functional calculus : the classes $H^\infty$ and $H_T^\infty$ . . . . .	109
3. The role of the outer functions in the functional calculus . . . . .	118
4. Contractions of class $C_\theta$ and their minimal functions . . . . .	122

5. Relations between minimal function and spectrum . . . . .	126
6. Relations between minimal function and invariant subspaces . . . . .	129
7. Characteristic vectors of a contraction of class $C_0$ and unicellularity . . . . .	134
8. Continuous one parameter semi-groups of contractions; their generators and co-generators . . . . .	140
9. Unitary dilation of semi-groups. Semi-groups of isometries . . . . .	145
<i>Notes</i> . . . . .	152

#### CHAPTER IV FUNCTIONAL CALCULUS: FUNCTIONS BOUNDED OR NOT

1. Calculation rules . . . . .	155
2. Representation of $\varphi(T)$ as a limit of $\varphi_r(T)$ . . . . .	161
3. Functions limited by a sector . . . . .	163
4. Accretive and dissipative operators . . . . .	167
5. Fractional powers of maximal accretive operators . . . . .	173
<i>Notes</i> . . . . .	180

#### CHAPTER V OPERATOR VALUED ANALYTIC FUNCTIONS

1. The spaces $L^2(\mathfrak{A})$ and $H^2(\mathfrak{A})$ . . . . .	183
2. Operator valued analytic functions. Inner and outer functions . . . . .	186
3. Lemmas on Fourier representations. Invariant subspaces for unilateral shifts . . . . .	192
4. Factorizations . . . . .	200
5. Existence of non-trivial factorizations for contractive analytic functions . . . . .	210
6. Contractive analytic functions with scalar multiples . . . . .	216
7. Factorization theorems for functions with scalar multiple . . . . .	227
8. Analytic kernels . . . . .	231
<i>Notes</i> . . . . .	233

#### CHAPTER VI CHARACTERISTIC FUNCTIONS AND FUNCTIONAL MODELS

1. Characteristic functions . . . . .	237
2. Functional models for a given contraction . . . . .	241
3. Functional models for given contractive analytic functions . . . . .	248
4. The characteristic function and the spectrum . . . . .	259
5. The characteristic function and the minimal function . . . . .	265
6. Spectral type of the minimal unitary dilation . . . . .	271
<i>Notes</i> . . . . .	276

## CHAPTER VII

REGULAR FACTORIZATIONS OF THE CHARACTERISTIC  
FUNCTION AND INVARIANT SUBSPACES

1. The fundamental theorem . . . . .	281
2. Some additional propositions . . . . .	289
3. Regular factorizations . . . . .	294
4. Arithmetic of regular divisors . . . . .	303
5. Invariant subspaces for contractions of class $C_{11}$ . . . . .	310
6. Spectral decomposition of contractions of class $C_{11}$ whose characteristic function admits a scalar multiple . . . . .	316
<i>Notes</i> . . . . .	319

## CHAPTER VIII

## WEAK CONTRACTIONS

1. Existence of a scalar multiple for the characteristic function . . . . .	323
2. Decomposition $C_0—C_{11}$ . . . . .	327
3. Spectral decomposition of weak contractions . . . . .	334
<i>Notes</i> . . . . .	340

## CHAPTER IX

PROBLEMS OF SIMILARITY, QUASI-SIMILARITY,  
AND UNICELLULARITY

1. Contractions which are similar to unitary operators . . . . .	341
2. Quasi-similarity of contractions . . . . .	345
3. Criteria of unicellularity . . . . .	350
4. Dissipative operators. Class $(\Omega_0^+)$ . . . . .	356
5. Dissipative operators similar to self-adjoint ones . . . . .	364
<i>Notes</i> . . . . .	368

REFERENCES . . . . .	373
INDEX . . . . .	385
NOTATIONS . . . . .	388