

# CONTENTS

## CHAPTER 1

<b>METRIC SPACES</b> .....	1
1. Set-theoretic Notation .....	1
2. Metric Spaces .....	2
3. Elementary Topology of Metric Spaces .....	5
4. Convergence and Completeness .....	7
5. The Contraction Theorem .....	10
6. Application of the Contraction Theorem to Differential Equations .....	13
7. The Baire Category Theorem .....	16
8. The Existence of a Continuous Nowhere-differentiable Function .....	18
9. Continuous Functions .....	21
10. Compactness .....	24

## CHAPTER 2

<b>LEBESGUE MEASURE AND INTEGRATION</b> .....	29
1. Introductory Remarks .....	29
2. Measure of Open Sets .....	31

3. Measure of More General Sets .....	36
4. Measurable Functions .....	44
5. Integration of Non-negative Functions .....	49
6. Integration of Real-valued and Complex-valued Functions .....	54
7. Integration over Plane Sets .....	57
8. Concluding Remarks .....	60

## CHAPTER 3

<b>THE <math>L^p</math>- AND <math>l^p</math>-SPACES</b> .....	62
1. Basic Concepts .....	62
2. The Hölder and Minkowski Inequalities .....	64
3. Definition of a Metric in $L^p$ .....	66
4. Completeness of $L^p$ .....	67
5. The Space $L^\infty$ .....	69
6. The Spaces $l_n^p$ and $l^p$ .....	70
7. Separability of $L^p$ and $l^p$ .....	73

## CHAPTER 4

<b>NORMED LINEAR SPACES</b> .....	76
1. Linear Spaces .....	76
2. Normed Linear Spaces .....	80
3. Inner-product Spaces .....	87
4. Hilbert Spaces .....	92
5. Orthonormal Bases in Hilbert Spaces .....	97

## CHAPTER 5

<b>LINEAR FUNCTIONALS</b> .....	102
1. Basic Definitions and Concepts .....	102
2. The Principle of Uniform Boundedness .....	104
3. Bounded Linear Functionals in Hilbert Spaces .....	109
4. The Dual Space of $L^p(A)$ .....	116
5. The Hahn-Banach Theorem .....	120

## CHAPTER 6

<b>OPERATORS</b> .....	126
1. Linear Transformations and Operators .....	126
2. The Adjoint Operator .....	129
3. The Inverse of an Operator .....	131

4. Sequences of Operators .....	135
5. Hermitian Operators .....	137
6. Projections .....	143
7. The Spectrum of an Operator .....	146
8. Spectra of Hermitian, Normal, and Unitary Operators ...	149

## CHAPTER 7

**OPERATORS ON FINITE-DIMENSIONAL SPACES** ..... 153

1. Matrix Representation of Linear Transformations .....	153
2. Eigenvalues and Eigenvectors .....	157
3. Finite-dimensional Inner-product Spaces .....	162
4. Kellogg's Method of Estimating the Largest Eigenvalue ..	168
5. Spectral Representation of Hermitian Operators .....	171
6. Spectral Representation of Normal Operators .....	174

## CHAPTER 8

**ELEMENTS OF SPECTRAL THEORY IN INFINITE-DIMENSIONAL HILBERT SPACES** ..... 177

1. Completely Continuous Operators .....	177
2. Spectral Analysis of a Hermitian Completely Continuous Operator .....	181
3. The Fredholm Alternative .....	185
4. Survey of the Fredholm Theory of Integral Equations ...	189
5. Estimation of the Largest Eigenvalue .....	193

**APPENDICES**

A. PARTIALLY ORDERED SETS AND ZORN'S LEMMA .....	197
B. CONCERNING THE SPECTRUM OF AN OPERATOR ON A COM- PLEX BANACH SPACE .....	200
C. THE STIELTJES INTEGRAL .....	201
D. THE WEIERSTRASS APPROXIMATION THEOREM AND AP- PROXIMATION BY TRIGONOMETRIC POLYNOMIALS ....	203
E. THE STRUCTURE OF OPEN SETS OF REAL NUMBERS ....	207
F. INFINITE SERIES AND THE NUMBER SYSTEM $[0, +\infty]$ ...	208

**x CONTENTS**

G. LIMIT SUPERIOR AND LIMIT INFERIOR ..... 211

H. THE FOURIER TRANSFORM IN  $L^2(\mathbb{R})$  ..... 214

**SOME SUGGESTIONS FOR FURTHER READINGS** ..... 221

**CITED REFERENCES** ..... 223

**INDEX** ..... 225