

Contents

Preface	XI
Introduction	1

Chapter 1

TOPOLOGICAL VECTOR SPACES

1.1. <i>Vector spaces</i>	7
1.1.1. General notions on vector spaces	7
1.1.2. Balanced sets, absorbent sets, convex sets	10
1.1.3. Linear functionals	12
1.1.4. Linear manifolds	14
1.1.5. Sublinear functionals and extension of linear functionals	15
1.2. <i>Topological vector spaces</i>	20
1.2.1. Definitions and general properties	20
1.2.2. Product spaces and quotient spaces	23
1.2.3. Bounded and totally bounded sets	25
1.2.4. Convex sets and compact sets in topological vector spaces	27
1.2.5. Closed hyperplanes and separation of convex sets	30
1.2.6. Complete topological vector spaces	32
1.3. <i>Metrizable topological vector spaces and normed vector spaces</i>	37
1.3.1. Metrizable topological vector spaces	37
1.3.2. Normed vector spaces	43
1.3.3. Normable topological vector spaces and finite-dimensional spaces	45
1.3.4. Pre-Hilbert spaces	48
1.3.5. Hilbert spaces	51
1.3.6. Supplements	55

Chapter 2

*INTRODUCTION TO THE THEORY OF LOCALLY
CONVEX SPACES*

2.1. <i>Locally convex spaces</i>	57
2.1.1. General properties	57
2.1.2. Subspaces, product spaces, quotient spaces	61
2.1.3. Convex and compact sets in locally convex spaces	64
2.1.4. Inductive limits of locally convex spaces	66
2.1.5. Topological direct sums and inductive spectra	69
2.1.6. Projective spectra	70
2.1.7. Bornological spaces	71
2.1.8. Barreled spaces	73
2.2. <i>Spaces of numerical functions</i>	74
2.2.1. Spaces of continuous functions	74
2.2.2. Spaces of indefinitely differentiable functions	75
2.2.3. The notion of distribution	77
2.2.4. Supplements	78

Chapter 3

*CONTINUOUS LINEAR OPERATORS.
SPACES OF OPERATORS. DUAL VECTOR SPACES*

3.1. <i>Continuous linear operators</i>	80
3.1.1. General considerations on continuous linear operators	80
3.1.2. Open operators and closed operators	82
3.1.3. Supplement	84
3.2. <i>Spaces of operators</i>	84
3.2.1. Topologies of uniform convergence	84
3.2.2. Properties of the spaces of continuous linear operators	86
3.3. <i>Dual vector spaces</i>	90
3.3.1. Dual spaces	90
3.3.2. The Mackey topology	94
3.3.3. The strong topology	96
3.3.4. Semireflexive spaces and reflexive spaces	97

3.3.5. Vector sequence spaces	98
3.3.6. Adjoint operators	102
3.3.7. Supplements	104
3.4. Tensor products	105
3.4.1. General considerations on tensor products	105
3.4.2. Topological tensor products	107
3.4.3. Supplements	111

Chapter 4

SPECIAL TYPES OF LOCALLY CONVEX SPACES

4.1. Nuclear spaces	112
4.1.1. General considerations	112
4.1.2. Properties of nuclear spaces	114
4.1.3. Supplements	116
4.2. Other types of locally convex spaces	117
4.2.1. Montel spaces	117
4.2.2. Schwartz spaces	118
4.2.3. (DF)-spaces and Silva spaces	120

Chapter 5

INTRODUCTION TO THE THEORY OF ORDERED VECTOR SPACES

5.1. General considerations on ordered vector spaces	125
5.1.1. Definitions and remarks	125
5.1.2. Quasi-Archimedean spaces and Archimedean spaces	127
5.1.3. The Riesz condition	128
5.2. Notions of the vector lattice theory	130
5.2.1. Definitions and general properties	130
5.2.2. The Dedekind extension	133
5.2.3. Convergence with respect to the order relation	135
5.2.4. Solid sets in vector lattices	137
5.2.5. Vector sublattices and quotient spaces	139

Topological vector spaces

- 5.2.6. Bands and components 140
- 5.2.7. Discrete, atomic, diffuse elements 144
- 5.2.8. Regular spaces 147
- 5.3. *Operators* 149
 - 5.3.1. Sublinear operators and linear operators with values in an ordered vector space 149
 - 5.3.2. Positive linear operators 150
 - 5.3.3. (o) -bounded operators and regular operators 152
 - 5.3.4. (o) -continuous operators 154

Chapter 6

LOCALLY FULL TOPOLOGIES ON ORDERED VECTOR SPACES

- 6.1. *Topologies on ordered vector spaces* 156
 - 6.1.1. Locally full topologies 156
 - 6.1.2. Extension of a locally full topology 159
 - 6.1.3. Topologies with respect to which the positive cone is closed 159
 - 6.1.4. Locally full topologies defined by norms 161
 - 6.1.5. Spaces in duality. Dual of an ordered locally convex space 165
 - 6.1.6. (o) -barreled topologies 170
- 6.2. *Topologies generated by the order structure* 171
 - 6.2.1. The topology of (o) -boundedness 171
 - 6.2.2. The topology of uniform convergence on (o) -bounded sets 173
 - 6.2.3. Spaces possessing a basis of positive elements 174
 - 6.2.4. Supplements 176

Chapter 7

LOCALLY SOLID TOPOLOGIES ON VECTOR LATTICES

- 7.1. *Topological vector lattices* 178
 - 7.1.1. Locally solid topologies on directed vector spaces 178
 - 7.1.2. Extension of a locally solid topology 180
 - 7.1.3. Locally solid topologies on vector lattices 180
 - 7.1.4. Solid seminorms on vector lattices 184
 - 7.1.5. The locally solid topology associated to a locally full topology 187
 - 7.1.6. The dual of a locally convex lattice 188

7.1.7. Vector lattices in (σ)-duality	193
7.1.8. The extension of a locally convex lattice	194
7.1.9. Certain types of locally convex lattices	196
7.1.10. Nuclear locally convex lattices	200
7.1.11. The topology of (σ)-boundedness on vector lattices	201
7.2. Normed lattices	203
7.2.1. Solid norms on vector lattices	203
7.2.2. Spaces of (KB)-type and spaces of (M)-type	205
7.2.3. Supplements	208

Chapter 8

OPERATORS BETWEEN TOPOLOGICAL ORDERED VECTOR SPACES

8.1. Linear operators and ordered spaces of operators	209
8.1.1. Existence and extension of certain continuous and positive linear operators	209
8.1.2. Continuity of certain positive linear operators	212
8.1.3. Operators with values in normed lattices	213
8.1.4. Spaces of continuous linear operators	217
8.1.5. Supplements	220
8.2. Bilinear operators and ordered tensor products	221
8.2.1. Ordered tensor products	221
8.2.2. Topological ordered tensor products	222
8.2.3. Supplements	224
References	225
Subject Index	229