

CONTENTS

<i>PART I</i>	<i>INTRODUCTION TO THE MULTIPLICATION OF DISTRIBUTIONS</i>	1
<i>Chapter 1</i>	<i>A Review of the Classical Heuristic Computations of Quantum Field Theory</i>	5
§ 1.1	– A brief historical survey	6
§ 1.2	– A Fock space	8
§ 1.3	– A free field	10
§ 1.4	– An interacting field equation	14
§ 1.5	– The canonical Hamiltonian formalism	16
§ 1.6	– Interacting field operators	18
§ 1.7	– The scattering operator	21
§ 1.8	– Wick products of free fields operators	28
<i>Chapter 2</i>	<i>A Review of Some Products of Distributions</i>	31
§ 2.1	– The method of regularization and passage to the limit	31
§ 2.2	– The Fourier transform method	36
§ 2.3	– Comparison of the two methods	38
§ 2.4	– Schwartz's impossibility result	46
<i>Chapter 3</i>	<i>A General Multiplication of Distributions</i>	49
§ 3.1	– Description of the basic idea	50
§ 3.2	– A concept of partial derivatives generalizing derivation in the distribution sense	53
§ 3.3	– A property of all C^∞ functions over $E^1(\Omega)$	55
§ 3.4	– Construction of the algebra $G(\Omega)$	60
§ 3.5	– Connection with classical products	66
§ 3.6	– Non linear functions of elements of $G(\Omega)$	73

PART II	A GENERALIZED MATHEMATICAL ANALYSIS	79
Chapter 4	Concepts of Generalized Functions	83
§ 4.1	– The use of germs of C^∞ functions on open sets of $D(\Omega)$	84
§ 4.2	– Another concept of generalized functions: The algebra $G_S(\Omega)$	93
§ 4.3	– Examples of solutions of differential equations	99
§ 4.4	– Some useful results	103
§ 4.5	– Local generalized functions	107
§ 4.6	– Restrictions of generalized functions to subspaces	111
§ 4.7	– Global weak solutions of ordinary differential equations	118
§ 4.8	– Examples of modifications of the concepts of generalized functions	127
§ 4.9	– Modifications of the sets A_q and “Removal of divergences”	129
Chapter 5	Pointvalues of Generalized Functions	135
§ 5.1	– Construction of the algebra $\bar{\mathbb{C}}$	136
§ 5.2	– Nonlinear properties of $\bar{\mathbb{C}}$	139
§ 5.3	– Point values of generalized functions	141
§ 5.4	– Examples	146
§ 5.5	– Constant generalized functions	149
§ 5.6	– Dependence of $\bar{\mathbb{C}}$ on the space dimension	151
Chapter 6	Integration of Generalized Functions	155
§ 6.1	– The integral of a generalized function on a compact set	155
§ 6.2	– Connections with classical integrals	160
§ 6.3	– Primitives	164
§ 6.4	– Convolution	167
§ 6.5	– Fourier transform	177
Chapter 7	Generalized Functions as Boundary Values of Usual C^∞ Functions	181
§ 7.1	– A simplification in the concepts of generalized functions	182
§ 7.2	– Pointvalues	192
§ 7.3	– Integration	195
§ 7.4	– Convolution	197
§ 7.5	– Generalized functions considered as “generalized distributions”	200
§ 7.6	– Fourier transform	201
§ 7.7	– Extension of domains and simplifications	203
§ 7.8	– Composition of generalized functions	207

Chapter 8	<i>Holomorphic Generalized Functions</i>	215
§ 8.1	– Integrals along curves	216
§ 8.2	– Definition and first examples of holomorphic generalized functions	217
§ 8.3	– Generalized Cauchy formulas	220
§ 8.4	– Characterization of holomorphic generalized functions as boundary values of usual holomorphic functions	224
§ 8.5	– Taylor and Laurent series expansions	228
§ 8.6	– Real analytic representatives	230
§ 8.7	– Analytic continuation	231
§ 8.8	– Holomorphic generalized functions of several complex variables	236
Chapter 9	<i>Further Concepts of Generalized Functions: Test Functions in $S(\mathbb{R}^n)$</i>	239
§ 9.1	– C^∞ functions on $S'(\mathbb{R}^n)$	239
§ 9.2	– Concept of generalized functions with $A_q \subset S(\mathbb{R}^n)$ simplified case	244
§ 9.3	– The algebras $G_1(\Omega)$, $G_{1,s}(\Omega)$ and their properties	248
§ 9.4	– The algebras $G_{1,\text{loc}}(\Omega)$, $G_{1,s,\text{loc}}(\Omega)$ and their properties	256
§ 9.5	– The algebras $G_1^*(\Omega)$ and $G_{1,s}^*(\Omega)$	260
§ 9.6	– Pointvalues, integration and applications	262
Chapter 10	<i>Further Concepts of Generalized Functions: Unbounded Sets A_q</i>	269
§ 10.1	– Motivations	269
§ 10.2	– Concepts of generalized functions	272
§ 10.3	– Pointvalues, integration and applications	276
§ 10.4	– Basic test functions in $S(\mathbb{R}^n)$	278
PART III	<i>A MATHEMATICAL SETTING FOR QUANTUM FIELD THEORY</i>	281
Chapter 11	<i>Vector Valued Generalized Functions and Free Fields</i>	285
§ 11.1	– Generalized functions valued in a bornological algebra	286
§ 11.2	– Pointvalues, integration and applications	292
§ 11.3	– The free fields operators are vector valued generalized functions	295
§ 11.4	– The Hamiltonian formalism of the free fields	299
§ 11.5	– Computation of the free fields Hamiltonian and removal of a first divergence	303

Chapter 12	<i>The Interacting Fields</i>	309
§ 12.1	– Self adjointness of some operators	311
§ 12.2	– Weak solutions of the interacting fields equations	313
§ 12.3	– Scattering operators	323
§ 12.4	– Wick products of the free fields	329
§ 12.5	– A glimpse at renormalization theory	332
§ 12.6	– A regularization of the free field operators and applications	343
§ 12.7	– Scattering operators (another method)	350
§ 12.8	– Asymptotic expansions and scattering operators	354
	<i>Bibliographic Notes</i>	363
	<i>References</i>	367
	<i>Index</i>	371