

# Contents

## Chapter 1

### Basic Properties of Hilbert Spaces

- 1.0 Introduction 1
- 1.1 Basic definitions 2
- 1.2 Examples of Hilbert spaces 5
- 1.3 Hilbert spaces from Hilbert spaces 6
- 1.4 Convex sets and projections 9
- 1.5 Orthogonality and orthonormal bases 14
- 1.6 Continuous linear functionals 19
- 1.7 Riesz representation theorem
- 1.7 Riesz representation theorem 19
- 1.8 Weak convergence 24
- 1.9 Nonlinear functionals and generalized curves 31
- 1.10 The Hahn–Banach theorem 37

## Chapter 2

### Convex Sets and Convex Programming

- 2.0 Introduction 38
- 2.1 Elementary notions 38
- 2.2 Support functional of a convex set 40
- 2.3 Minkowski functional 42
- 2.4 The support mapping 46
- 2.5 Separation theorem 47
- 2.6 Application to convex programming 50
- 2.7 Generalization to infinite dimensional inequalities 52
- 2.8 A fundamental result of game theory: minimax theorem 54
- 2.9 Application: theorem of Farkas 58

## Chapter 3

## Functions, Transformations Operators

- 3.0 Introduction 62
- 3.1 Linear operators and their adjoints 63
- 3.2 Spectral theory of operators 80
- 3.3 Spectral theory of compact operators 91
- 3.4 Operators on separable Hilbert spaces 97
- 3.5  $L_2$  spaces over Hilbert spaces 133
- 3.6 Multilinear forms 146

## Chapter 4

## Semigroups of Linear Operators

- 4.0 Introduction 162
- 4.1 Definitions and general properties of semigroups 163
- 4.2 Generation of semigroups 170
- 4.3 Semigroups over Hilbert spaces: dissipative semigroups 173
- 4.4 Compact semigroups 176
- 4.5 Analytic (holomorphic) semigroups 182
- 4.6 Elementary examples of semigroups 188
- 4.7 Extensions 196
- 4.8 Differential equations: Cauchy problem 202
- 4.9 Controllability 207
- 4.10 State reduction: observability 210
- 4.11 Boundary input: an example 214
- 4.12 Evolution equations 220

## Chapter 5

## Optimal Control Theory

- 5.0 Introduction 226
- 5.1 Preliminaries 227
- 5.2 Linear quadratic regulator problem 228
- 5.3 Linear quadratic regulator problem: infinite time interval 233
- 5.4 Hard constraints 239
- 5.5 Final value control 243
- 5.6 Time optimal control problems 248

## Chapter 6

## Probability Measures on a Hilbert Space

- 6.0 Introduction 252
- 6.1 Preliminaries 253
- 6.2 Measures on cylinder sets 256
- 6.3 Characteristic functions and countable additivity 264
- 6.4 Weak random variables 265
- 6.5 Random variables 271

## Contents

- 6.6 White noise 272
- 6.7 Differential systems drive by white noise 273
- 6.8 The filtering problem 276
- 6.9 Stochastic control 285
- 6.10 White noise integrals 293
- 6.11 Radon–Nikodym derivatives 300

**Bibliography** 305

**Index** 307