Table of Contents

Editor's Preface ix

Introduction xiii

CHAPTER 1. *Metric spaces* 1
1.1. Definition and examples of metric spaces 1
1.2. Convergence and related notions 9
1.3. Continuous mappings 13
1.4. Semimetric spaces 16
1.5. Completeness of metric spaces 17
1.6. Contractive mapping principle 23
1.7. Sets of first and second category 27
1.8. Spaces of modulus integrable and square integrable functions 29
1.9. Basic facts of measure theory and the Lebesgue integral 32
1.10. Separable spaces 45
1.11. Sequentially compact spaces and compact spaces . . . 49

CHAPTER 2. *Linear spaces and normed spaces* 55
2.1. Basic facts about linear spaces 55
2.2. Linear metric spaces and normed spaces 62
2.3. Linear functionals 77
2.4. Finite-dimensional spaces 85
2.5. Extensions of functionals 90
2.6. General forms of continuous linear functionals in certain spaces 100

CHAPTER 3. *Continuous linear operators in Banach spaces* . . . 114
3.1. The Banach–Steinhaus theorem 114

3.2. The Banach theorem about the continuity of the inverse operator 118
3.3. Closed operators 122
3.4. Conjugate operators 133

CHAPTER 4. *Weak topologies* 138
4.1. Why we introduce topological spaces? Basic topological notions 138
4.2. Sequentially compact spaces and compact spaces 143
4.3. Linear topological spaces 146
4.4. Weak topologies 149
4.5. Reflexive spaces and weak compactness 156
4.6. Extreme points 159

CHAPTER 5. *Optimization and observability of linear systems* . . 165
5.1. Linear systems 165
5.2. Optimization of linear systems with a fixed input operator 179
5.3. Sufficient conditions for the existence of an optimal input 201
5.4. Minimum-time problem 208
5.5. Reduction of the minimum-time problem to the minimum-norm problem 225
5.6. Observability of linear systems 231
5.7. Minimum-time problems of observability theory 249
5.8. Noncontinuous linear systems. Estimability, observability, controllability 253

CHAPTER 6. *Systems governed by ordinary differential equations* 262
6.1. Problems of minimization of convex functionals related to systems governed by ordinary differential equations 262
6.2. Controllability of finite-dimensional systems 267
6.3. Minimum-norm problems for sup type norms 269
6.4. Criteria for the uniqueness of optimal control 274
6.5. Bang-bang principle 277
6.6. Measurable multifunctions and their applications . . . 285
6.7. Observability of systems described by ordinary differential equations 300

6.8. Optimal observation of stationary systems 316
6.9. Functionals observed by less than n measurements . . 318
6.10. Optimal integration and differentiation formulae 322

CHAPTER 7. *Systems with distributed parameters* 328
7.1. Bases in Banach spaces 329
7.2. Eigenvalues and eigenvectors 340
7.3. Temperature distribution in a rod with zero boundary conditions 345
7.4. Nonhomogeneous heat diffusion equation with zero boundary conditions 361
7.5. Homogeneous heat diffusion equation with nonhomogeneous boundary conditions 366
7.6. Rod heating control 376
7.7. Observability of temperature distribution in a rod . . . 384
7.8. Other problems related to heat diffusion in one-dimensional objects 394
7.9. Rod vibration control 403

CHAPTER 8. *Differential and integral equations in Banach spaces* 418
8.1. Derivatives and integrals of functions with values in Banach spaces 418
8.2. Volterra integral equations and differential equations with continuous right-hand sides 421
8.3. Autonomous differential equation with a discontinuous right-hand side and semigroups of operators 428
8.4. Properties of the infinitesimal generator 440
8.5. Infinitesimal generator of bounded semigroups 445
8.6. Nonautonomous differential equations in Banach spaces 449
8.7. Controllability and approximative controllability of linear systems described by differential equations in Banach spaces 452
8.8. Long-term controllability and long-term approximative controllability 463
8.9. Linear systems with analytic semigroups 467

8.10. Optimization problems for systems described by linear differential equations in Banach spaces 472
8.11. Duality between observability and controllability of systems governed by autonomous linear differential equations in Banach spaces 476
8.12. Stability of solutions of linear differential equations in Banach spaces 481

APPENDIX. *Necessary and sufficient condition for weak duality* 495

Bibliography 505

Author index 513

Subject index 515

List of symbols 523